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A subset of the sample space is called an **event**

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The event "a 4 is drawn" is a subset with four elements.

# Probability Axioms

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**Axiom 2:** If $\Omega$ represents the entire sample space of an experiment,

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**Axiom 3:** If $A_1, A_2, A_3, \ldots$ is an infinite collection of **disjoint** events,

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = \sum_{i=1}^{\infty} P(A_i)$$

# Additional Properties

Events for which there are no corresponding outcomes have probability zero.

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The probability of an event cannot exceed 1. For any event $A$,

$$P(A) \leq 1$$

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The rule extends to unions of more than two events, but becomes more complicated:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$-P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

# Counting Rules for Ordered Pairs

Suppose an ordered pair

$$(P_i, Q_j)$$

is to be made up with $P_i$ chosen from a set of $n$ candidates:

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Then the number of distinct ordered pairs that can possibly result is:

$$n \times m$$

# Counting Rules for n-tuples

The rule extends to ordered triples: If

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is to be made up with $P_i$ chosen from a set of $n$ candidates, $Q_j$ from a set of $m$, and $R_k$ from a set of $o$, the number of distinct ordered pairs that can possibly result is:

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In general, if we are choosing an ordered list of $k$ elements with $n_1$ choices for the first element, $n_2$ for the second, and so on, the number of possible ordered $k - tuples$ is:

$$n_1 \cdot n_2 \cdot n_3 \cdots n_k$$

# Permutations

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The number of permutations (ordered subsets) of size $k$ taken from a set of $n$ objects is:

$$P_{k,n} = \frac{n!}{(n-k)!}$$

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$$C_{k,n} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

# Examples

A soccer league has 11 teams in division 1 and 10 in division 2. If the championship match always has one team from division 1 and one from division 2, how many different pair of teams are possible in the championship game?

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Answer: 110 (by the product rule for ordered pairs with $n_1 = 11$ and $n_2 = 10$, the number of pairs is $n_1 n_2 = 11 \cdot 10 = 110$)

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Answer: 6 (by the product rule for ordered pairs with $n_1 = 3$ and $n_2 = 2$, the number of pairs is $n_1 n_2 = 3 \cdot 2 = 6)$

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Answer: This will be the number of subsets containg three elements chosen from a set of 10, with order not important.

Because the order does not matter, we use **combinations**

$$C_{3,10} = \binom{10}{3} = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$$

You can use the spreadsheet function =COMBIN(10,3) to compute this.

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Answer: Use the more general form of the product rule, with $n_1 = 4$, $n_2 = 5$, and $n3 = 3$. The number of different meals is:

$$n_1 \cdot n_2 \cdot n_3 = 4 \cdot 5 \cdot 3 = 6$$

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Answer: This will be the number of subsets containg three elements chosen from a set of 420, with order important.

Because the order matters, we use **permutations**

$$P_{3,420} = \frac{420!}{417!} = 420 \cdot 419 \cdot 418 = 73,559,640$$

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