Discrete Distributions

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Another way to say this is that we take binomial random variables with larger and larger n, but we keep the *expected number of successes* $np = \lambda$ the same for all of them.

The limit of the distribution of such a sequence of random variables as $n \to \infty$ is a Poisson.

The Poisson Distribution

The probability mass function is:

$$f(x) = p(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

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Computation:

Value	R	Spreadsheet
$P(X = x)$	$dpois(x, \lambda)$	$= POISSON(x, \lambda, FALSE)$
$P(X \leq x)$	$ppois(x, \lambda)$	$= POISSON(x, \lambda, TRUE)$

The Poisson Distribution

Now we will perform some numerical experiments.

First generate a sample of $1,000,000$ observations for a poisson experiment with $\lambda = 4$:

```
x<-rpois(1000000,4)
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hist(x)

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The results through $X = 6$ should look something like:

0	1	2	3	4	5	
18391	72886	146819	195399	195578	156312	10398

The Poisson Distribution

0	1	2	3	4	5	
18391	72886	146819	195399	195578	156312	10398

Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

dpois(0,4)

The Poisson Distribution

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[1]   0.1831

The Poisson Distribution

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To get the probability that $X = 1$ enter

dpois(1,4)

The Poisson Distribution

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Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

dpois(0,4)

The result should be something like

[1] 0.1831

To get the probability that $X = 1$ enter

dpois(1,4)

This time the results should look something like:

[1] 0.07326

The Poisson Distribution

0	1	2	3	4	5	
18391	72886	146819	195399	195578	156312	10398

Next compute the probability that $X = 2$:

dpois(2,4)

The Poisson Distribution

0	1	2	3	4	5	
18391	72886	146819	195399	195578	156312	10398

Next compute the probability that $X = 2$:

dpois(2,4)

The result should be something like

[1]   0.146525

The Poisson Distribution

0	1	2	3	4	5	
18391	72886	146819	195399	195578	156312	10398

Next compute the probability that $X = 2$:

dpois(2,4)

The result should be something like

[1]   0.146525

To get the probability that $X = 5$ enter

dnbinom(5,3,0.4)

The Poisson Distribution

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18391	72886	146819	195399	195578	156312	10398

Next compute the probability that $X = 2$:

dpois(2,4)

The result should be something like

[1] 0.146525

To get the probability that $X = 5$ enter

dnbinom(5,3,0.4)

This time the results should look something like:

[1] 0.175467

The Poisson Distribution

The expected value $E(X)$ in this case is:

$$E(X) \;=\; \lambda \;=\; 4$$

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To compute the sample mean $\overline{x}$, enter

mean(x)

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To compute the sample mean $\overline{x}$, enter

mean(x) The result should be something like

[1] 3.999121

The Poisson Distribution

The variance $V(X)$ in this case is:

$$V(X) \; = \; \lambda = 4$$

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To compute the sample variance s^2, enter

var(x)

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[1] 3.999059

The Poisson Distribution

The number of cars arriving per minute at a toll booth has a poisson distribution, and the average number of cars arriving per minute is 12.

Find the probability that exactly 10 cars arrive in a certain minute.

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Solution: 0.104837

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$dpois(10, 12)$

The Poisson Distribution

The number of phone calls going through a certain exchange per second has a Poisson distribution with $\lambda = 6$

Find 8 or fewer calls arrive in a given second.

The Poisson Distribution

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Solution: 0.84723

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Solution: 0.84723

$ppois(8, 6)$

The Poisson Distribution

The number of gypsy moth egg masses per square yard of bark surface has a poisson distribution.

If the average number of masses per square yard is 3, find the probability that more than 6 egg masses are found in a give square yard.

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Solution: 0.083918

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If the average number of masses per square yard is 3, find the probability that more than 6 egg masses are found in a give square yard.

Solution: 0.083918

$1 - ppois(5, 3)$

The Poisson Distribution

The number of tadpoles per liter of pond water has a poisson distribution with a mean of 28.

Find the probability that a 1-liter sample has 40 or more tadpoles.

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Find the probability that a 1-liter sample has 40 or more tadpoles.

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$1 - ppois(39, 28)$

The Poisson Distribution

The number of deer ticks per square yard has a Poisson distribution with a mean of 12.

Find the probability that a certain square yard has fewer than 11 ticks.

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Solution: 0.65277

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$ppois(10, 12)$