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If n is the number of trials and p is the probability of success, the distribution of the number of successes is approximately normal with:

$$\text{mean } \mu = np \quad \text{and} \quad \sigma = \sqrt{n \cdot p(1-p)}$$

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Approximate this as a normal distribution with

$$\text{mean} = 1000 \cdot 0.63 = 630$$

and

$$\text{standard deviation} = \sqrt{1000 \cdot 0.63 \cdot 0.37} = 15.36$$

so the probability is $pnorm(610, 630, 15.36)$ which gives 0.0964

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Normal Approximation

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The formula for the previous example's solution is:

=NORMDIST(650,630,15.36,TRUE)
-NORMDIST(610,630,15.36,TRUE) which also gives 0.807

Percentiles

Suppose 63 percent of people in a large urban area actually support a certain political candidate. If a poll samples 1000 voters, what is the 75^{th} percentile of the number of supporters in the sample?

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$$\text{standard deviation} = \sqrt{np(1-p)} = \sqrt{1000 \cdot 0.63 \cdot 0.37} = 15.36$$

so the percentile is $qnorm(0.75, 630, 15.36)$ which is 640.

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Percentiles for the normal distribution can also be obtained from spreadsheets.

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For the previous example the formula would be:

=NORMINV(0.75,630,15.36) which is 640.

The Central Limit Theorem

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If $X_1, X_2, \ldots, X_n$ is a **random sample** from a $N(\mu, \sigma)$, then the following statement is true:

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