Set Theory Introduction

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An example of a well-defined set would be the set of all members of the class of 2012 at Stonehill on 1/1/2010.

An example of a set that is not well-defined would be the set of all healthy people.

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The easiest way to specify a set is to list its elements, usually enclosed in curly brackets $\{\}$. The set of positive integers less than 9 would be denoted by:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

In a list of elements, nothing is ever repeated; if the element is listed once, it belongs to the set, otherwise it does not.

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Still larger sets can be specified using *set builder notation*. The set of real numbers between three and five, inclusive, is

$$A = \{x : 3 \leq x \leq 5\}$$

read "A is the set of all x values such that $3 \leq x \leq 5$.

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$A = \{1, 3, 4, 5, 9\} \quad B = \{1, 2, 9, 10, 11\}$ then $A \cup B = \{1, 2, 3, 4, 5$

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Definition: set complement The **complement** of a set A, denoted by A' or A^c, is the set consisting of all elements that **do not** belong to A. Implicit in the definition is a *universal set* U, and the complement of A is the set of all elements of U that do not belong to A. In many applications it is obvious what the universal set is; otherwise, it must be specified.

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Suppose

$U = \{1, 2, 3, 4, \ldots\}$ and $A = \{1, 3, 5, 7, \ldots\}$ then $A' = \{2, 4, 6$

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As we will see, the null set $\emptyset$ is an extremely useful concept.

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Perhaps the most famous of these is the *Russell paradox* due to Bertrand Russell: Consider the set A consisting of all sets that do not contain themselves as an element. Does A contain itself?

Experiments and Outcomes

Definition: experiment The author defines an **experiment** as any action or process whose outcome is subject to uncertainty.

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Examples of experiments include tossing a coin, rolling a single die, drawing a card from a shuffled deck, having someone pick a number between one and ten, and so on.

Experiments and Outcomes

Definition: outcome An **outcome** is the result of performing an experiment.

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The coin toss experiment is considered to have two outcomes, heads and tails. The experiment consisting of rolling a die has six outcomes (one through six). Drawing a card has 52 outcomes.

Experiments and Outcomes

Definition: sample space The **sample space** $\mathcal{S}$ of an experiment is the set containing all possible outcomes of the experiment.

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The sample space of the coin toss experiment, abbreviating heads and tails as H and T respectively, is:

$$\mathcal{S} = \{H, T\}$$

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The sample space of the die rolling experiment is:

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The sample space of the card drawing experiment has 52 elements,

$$\mathcal{S} = \{A\heartsuit, K\heartsuit, \ldots, 2\heartsuit, A\diamondsuit, \ldots, 2\diamondsuit, A\clubsuit, \ldots, 2\clubsuit, A\spadesuit, \ldots, 2\spadesuit\}$$

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Both of these are compound events.