Sullivan Section 6.2

Gene Quinn

Binomial Probability Experiments

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- The probability of success is the same for each trial

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X is said to be a **random variable** having the **binomial distribution**.

If the probability of success is $1/2$, the binomial experiment is equivalent to a series of coin tosses.

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- The probability of success on each trial is denoted by p.
- The probability of failure on each trial is $1 - p$.
- The total number of successes in n independent trials is denoted by X.

Computing Binomial Probabilities

The following structure known as **Pascal's triangle** is useful for computing binomial probabilities when n is fairly small ($n < 10$).

								1							
$n=1$							1		1						
$n=2$						1		2		1					
$n=3$					1		3		3		1				
$n=4$				1		4		6		4		1			
$n=5$			1		5		10		10		5		1		
$n=6$		1		6		15		20		15		6		1	
$n=7$	1		7		21		35		35		21		7		1

Computing Binomial Probabilities

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$n = 2$					1		2		1				
$n = 3$				1		3		3		1			
$n = 4$			1		4		6		4		1		
$n = 5$		1		5		10		10		5		1	
$n = 6$	1		6		15		20		15		6		1

The entries in successive rows of Pascal's triangle are the sum of the two closest entries in the previous row.

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In the row of Pascal's triangle corresponding to n trials, there are $n + 1$ entries.

The sum of the entries in the row corresponding to n trials is always 2^n.

This represents the number of possible sequences of n letters where each one has to be either S or F.

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- The fourth entry is the number of sequences having 3 $S's$ and $n-3$ $F's$

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- The next to last entry is the number of sequences having $n-1\ S's$ and $1\ F$ (the next to last entry is always n)
- The last entry is the number of sequences having $n\ S's$ and $0\ F's$ (the last entry is always 1)

Binomial Probabilities when $p = 0.5$

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If we identify "heads" with "success", the experiment corrsponds tossing a fair coin n times.

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The simplest case occurs when success and failure are equally likely.

If we identify "heads" with "success", the experiment corrsponds tossing a fair coin n times.

In this case, the probability of obtaining 0, 1, 2, etc. heads in n tosses is the corresponding entry in Pascal's table, divided by the sum of the row (2^n).

Binomial Probabilities when $p \neq 0.5$

When success and failure are *not* equally likely, we need to use the following modified procedure to calculate the probabilities.

The number of trials n determines which row of Pascal's triangle is used.

Binomial Probabilities when $p \neq 0.5$

Suppose the probability of success on each trial is p.

We compute the probabilities associated with each value of X,
where X represents the number of successes in n trials.

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- Continue in this fashion. The $n+1^{st}$ entry is multiplied by $(1-p)^n$

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For convenience, we define $0!$ to be 1.

Computing Binomial Probabilities

Definition: The number of **combinations** of n objects taken r at a time is denoted by either

$${}_nC_r \qquad \text{or} \qquad \binom{n}{r}$$

and is defined to be:

$$\frac{n!}{r!(n-r)!}$$

Computing Binomial Probabilities

Example: Find the number of **combinations** of 4 objects taken 2 at a time.

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By definition,

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1) \cdot (2 \cdot 1)} = \frac{24}{(2)(2)} = 6$$

Computing Binomial Probabilities in G

The general formula for computing the probability of k successes in a binomial experiment with n trials when the probability of success on each trial is p is:

$$P(k \text{ successes}) \quad = \quad \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

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or, equivalently,

$$P(k \text{ successes}) =_n C_k p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

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Most people find it better to use a spreadsheet, for convenience and accuracy

The BINOMDIST Function

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We have to know the number of trials in the experiment, n

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BINOMDIST(4,10,0.6,FALSE)
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To find the probability that exactly 7 out of 10 trials are successes when $p = 0.6$, the formula would be:

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BINOMDIST(7,10,0.6,FALSE)
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To find the probability that 4 **or fewer** out of 10 trials are successes when $p = 0.6$, the formula would be:

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To find the probability that 7 or fewer out of 10 trials are successes when $p = 0.6$, the formula would be:

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BINOMDIST(7,10,0.6,TRUE)
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The BINOMDIST Function

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To find the probability that **at least** 8 out of 10 trials are successes when $p = 0.6$, the formula would be:

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To find the probability that **at least** 8 out of 10 trials are successes when $p = 0.6$, the formula would be:

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To find the probability of at least 8 successes, we add the probabilities of 1, 2, 3,...,7 successes and subtract the total from 1.

The BINOMDIST Function

In summary, for a binomial experiment with n trials and probability of success p, the probabilities of some common events are:

exactly k successes	`=BINOMDIST(k,n,p,FALSE)`
k or fewer successes	`=BINOMDIST(k,n,p,TRUE)`
at least k successes	`=1-BINOMDIST(k-1,n,p,TRUE)`
more than k successes	`=1-BINOMDIST(k,n,p,TRUE)`
fewer than k successes	`=BINOMDIST(k-1,n,p,TRUE)`
fewer than j or more than k successes	`=1+BINOMDIST(j-1,n,p,TRUE)` `-BINOMDIST(k,n,p,TRUE)`
Between j and k successes (inclusive)	`=BINOMDIST(k,n,p,TRUE)` `-BINOMDIST(j-1,n,p,TRUE)`

Mean of a Binomial Random Variable

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The possible outcomes of the experiment, and the probabilities associated with each outcome are completely determined by two numbers:

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Immediately, we know that the random variable X defined to be the number of successes obtained in the experiment must have one of the following values:

$$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6$$

Furthermore, we know that for $k = 0, 1, \ldots, 6$, the probability that exactly k successes are obtained is given by the formula:

$$P(X = k) \quad = \quad {}_6C_k \cdot p^k(1-p)^{n-k}$$

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Means and Standard Deviations

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The **population standard deviation** σ_X is given by the formula:

$$\sigma_X = \sqrt{n \cdot p \cdot (1 - p)}$$

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The standard deviation σ_X is given by

$$\sigma_X \;=\; \sqrt{n \cdot p \cdot (1-p)} \;=\; \sqrt{100 \cdot 0.6 \cdot 0.4} \;=\; 4.90$$

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One of the properties of the binomial probability distribution is that the distribution is bell shaped when n is reasonably large.

How large is a "reasonably large" value of n? It depends on p.

A commonly used rule of thumb states that the binomial distribution will be approximately bell shaped provided that

$$n \geq \frac{10}{p \cdot (1-p)}$$

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Earlier we found that for a binomial experiment with 100 trials each having a probability of 0.6 of success, the mean and standard deviation were:

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$n = 100$ is more than adequate to satisfy the rule of thumb stating that n should be greater than or equal to $10/(p \cdot (1-p))$, so the empirical rule tells us that:

- approximately 68% of the time X will fall in the range 55.1 to 64.9
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