The General Normal Distribution

Gene Quinn

The Standard Normal Distribution

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Various standard normal probabilities can be computed with the **NORMDSIST** function

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The standard deviation of a general normal distribution can be any positive number.

Henceforth, we will drop the word "general" and "normal distribution" will mean a normal or bellcurve distribution with arbitrary mean and standard deviation.

The Normal Distribution

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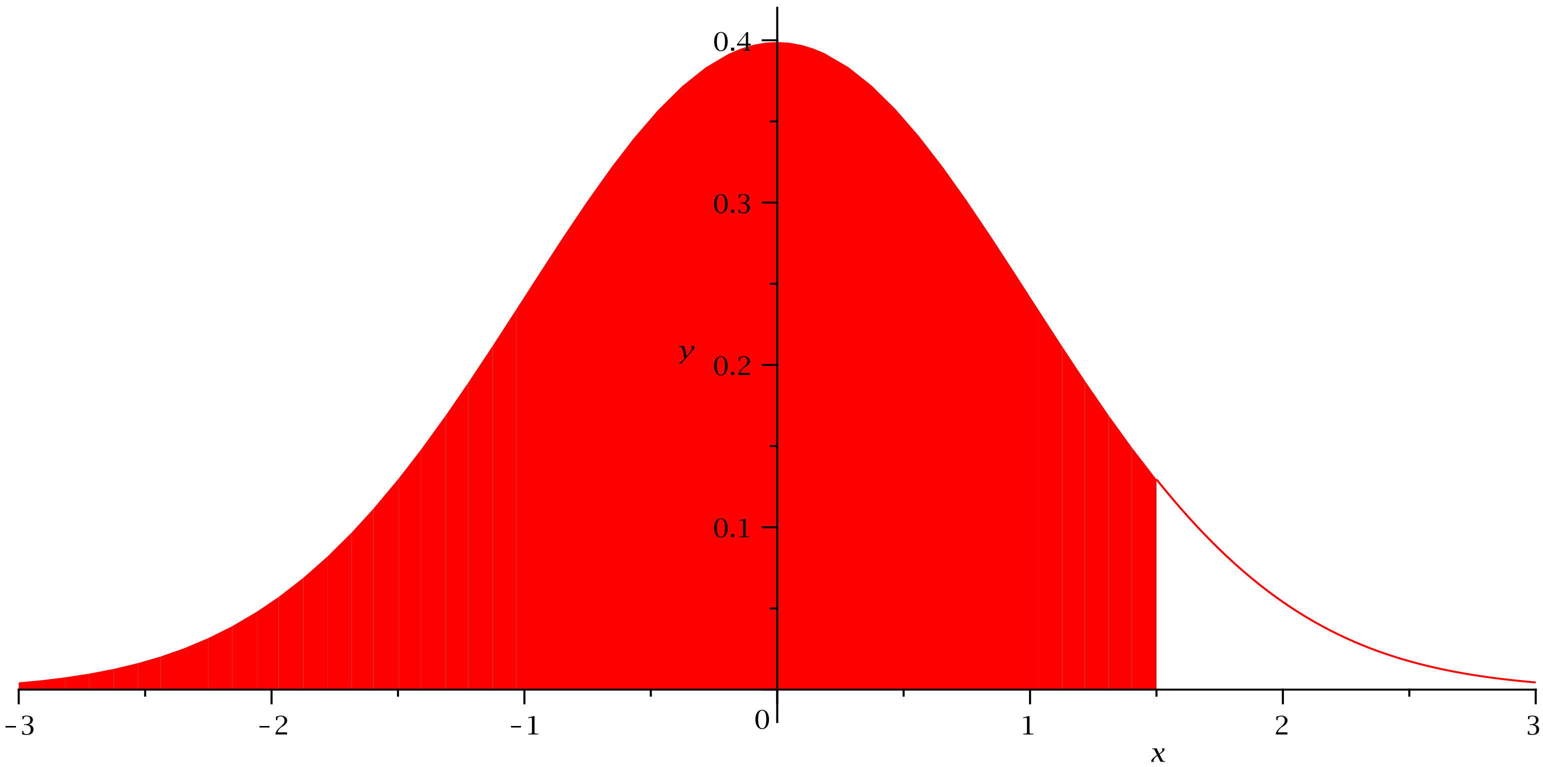

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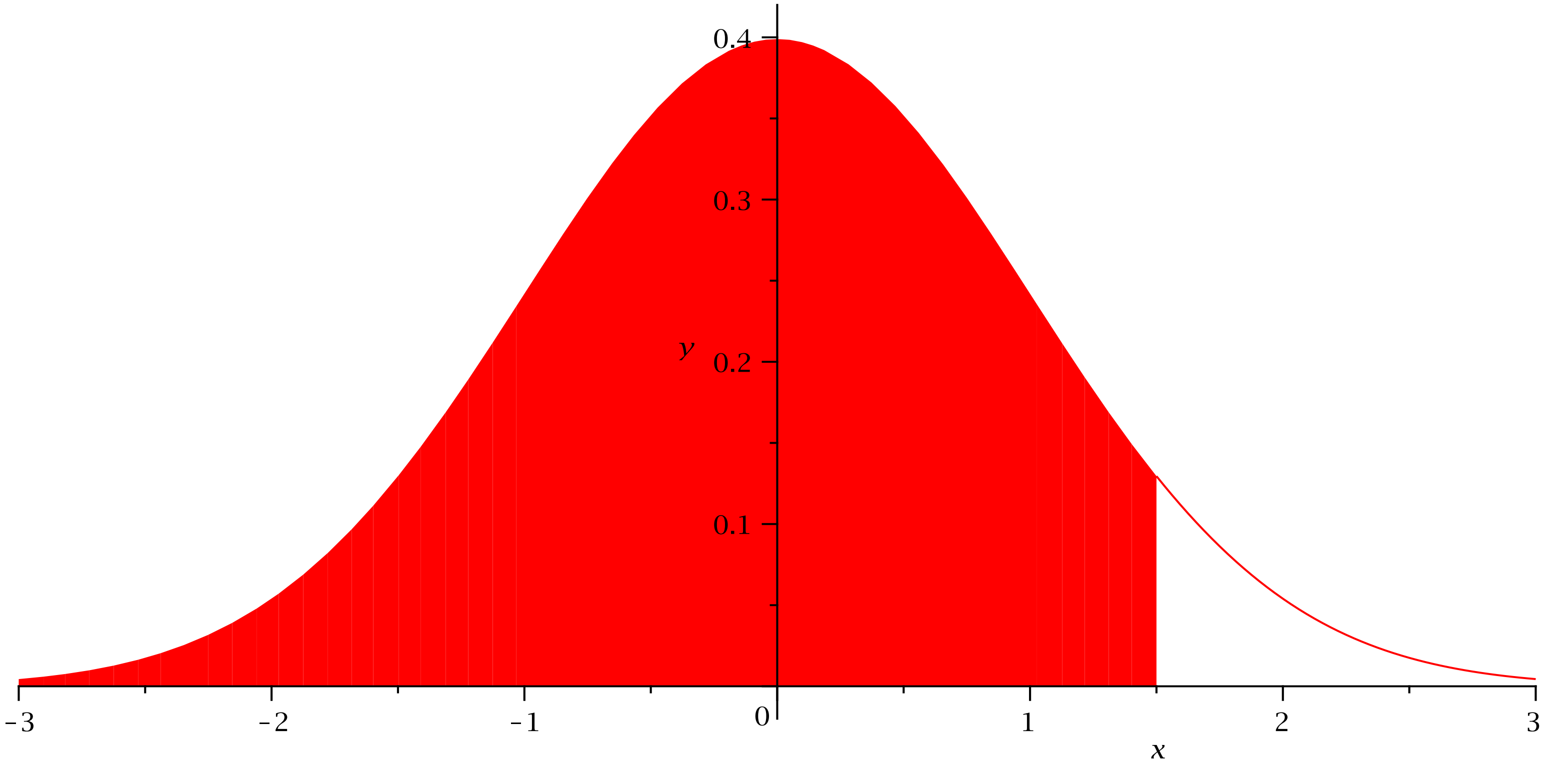

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The last argument will always be "TRUE" in our applications.

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The **NORMSDIST** function takes **four** arguments

The first is the value of X, the point on the horizontal axis (just like NORMSDIST)

The second and third are the mean and standard deviation.

We have to supply these because they are no longer assumend to be zero and one.

The last argument will always be "TRUE" in our applications.

=NORMDIST(X,mean,stdev,TRUE)

The Normal Distribution

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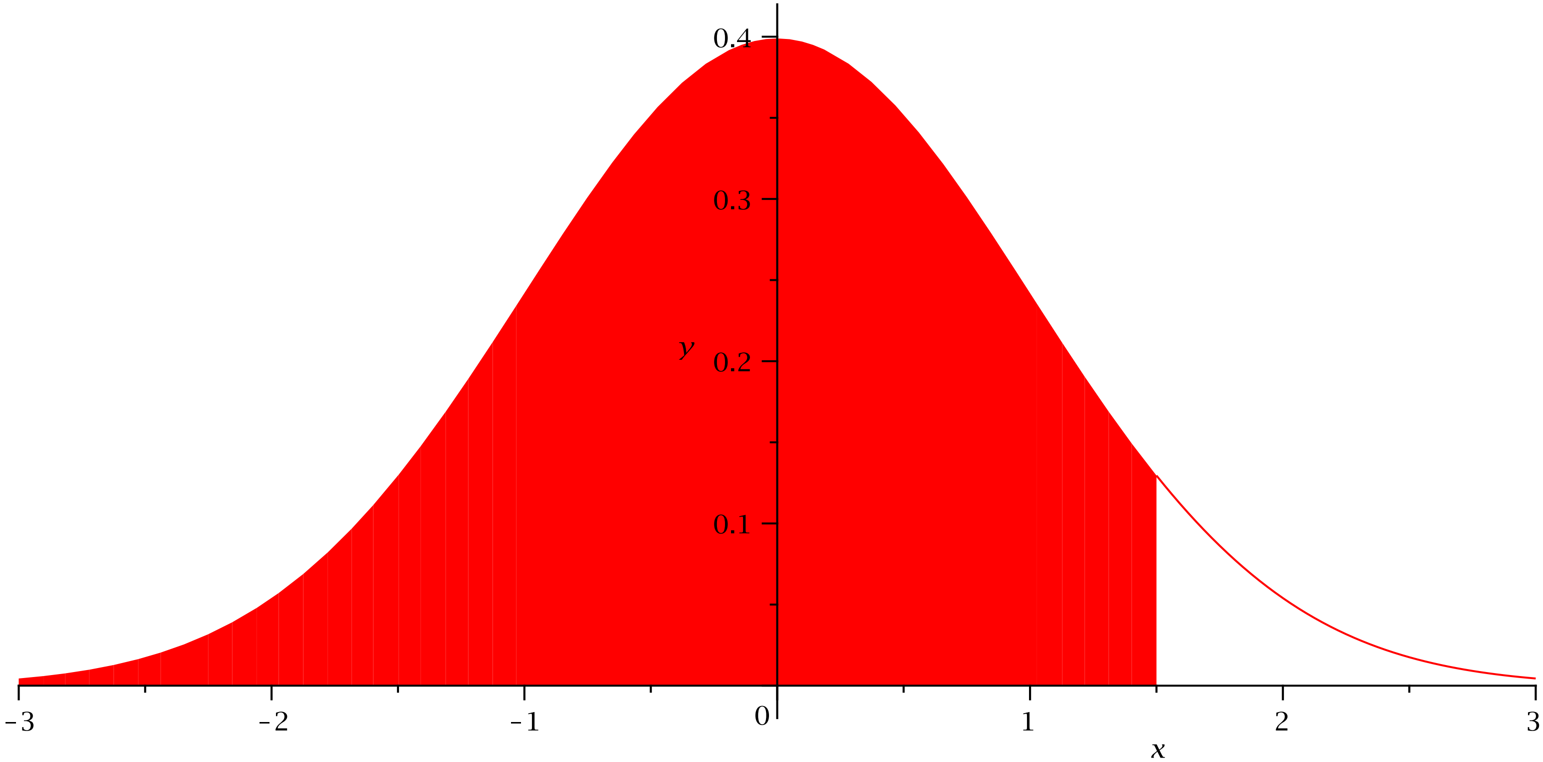

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores less than 110 on an IQ test.

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The result is 0.748

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores less than 110 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)**

The result is 0.748

This means that 74.8 percent of individuals score less than 110 on an IQ test.

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IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores less than 110 on an IQ test.

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This means that 74.8 percent of individuals score less than 110 on an IQ test.

It also means that a randomly selected individual has a probability of 0.748 of scoring less than 110.

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Find the probability that a randomly selected individual scores less than 95 on an IQ test.

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Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores less than 95 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)**

The result is 0.369

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores less than 95 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)**

The result is 0.369

This means that 36.9 percent of individuals score less than 110 on an IQ test.

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores less than 95 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)**

The result is 0.369

This means that 36.9 percent of individuals score less than 110 on an IQ test.

It also means that a randomly selected individual has a probability of 0.369 of scoring less than 95.

Example: Normal Distribution

The SAT is designed so that the mean of the scores is 500 and the standard deviation is 100.

Find the probability that a randomly selected individual scores less than 600 on the SAT.

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The SAT is designed so that the mean of the scores is 500 and the standard deviation is 100.

Find the probability that a randomly selected individual scores less than 600 on the SAT.

Enter **=NORMSDIST(110,100,15,TRUE)**

The result is 0.841

Example: Normal Distribution

The SAT is designed so that the mean of the scores is 500 and the standard deviation is 100.

Find the probability that a randomly selected individual scores less than 600 on the SAT.

Enter **=NORMSDIST(110,100,15,TRUE)**

The result is 0.841

This means that 84.1 percent of individuals score less than 600 on an SAT test.

Example: Normal Distribution

The SAT is designed so that the mean of the scores is 500 and the standard deviation is 100.

Find the probability that a randomly selected individual scores less than 600 on the SAT.

Enter **=NORMSDIST(110,100,15,TRUE)**

The result is 0.841

This means that 84.1 percent of individuals score less than 600 on an SAT test.

It also means that a randomly selected individual has a probability of 0.841 of scoring less than 600.

Example: Normal Distribution

Find the proportion of a normal population with mean 50 and standard deviation 10 that is less than 35.

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Enter **=NORMSDIST(35,50,10,TRUE)**. The result is 0.067

Example: Normal Distribution

Find the proportion of a normal population with mean -40 and standard deviation 25 that is less than zero.

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Find the proportion of a normal population with mean -40 and standard deviation 25 that is less than zero.

Enter **=NORMSDIST(0,-40,25,TRUE)**. The result is 0.945

The Normal Distribution

Sometimes we are interested in the probability that an observation from a normal population is **greater than** a given value.

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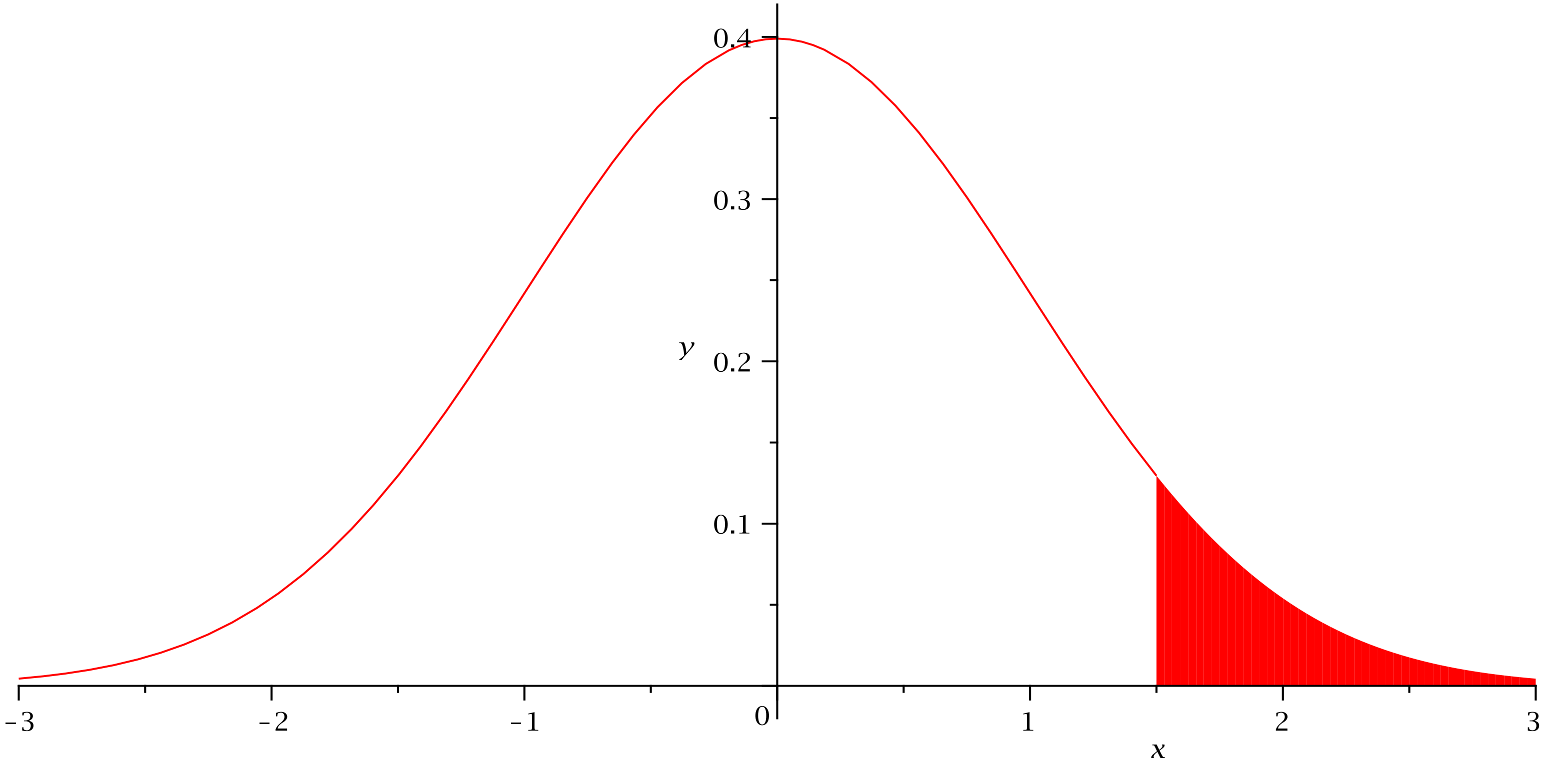
Sometimes we are interested in the probability that an observation from a normal population is **greater than** a given value.

The area to the **right** of x is given by
=1-NORMDIST(x,mean,stdev,TRUE)



Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores higher than 110 on an IQ test.

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Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores higher than 110 on an IQ test.

Enter **=1-NORMSDIST(110,100,15,TRUE)**

The result is 0.252

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores higher than 110 on an IQ test.

Enter **=1-NORMSDIST(110,100,15,TRUE)**

The result is 0.252

This means that 25.2 percent of individuals score higher than 110 on an IQ test.

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores higher than 110 on an IQ test.

Enter **=1-NORMSDIST(110,100,15,TRUE)**

The result is 0.252

This means that 25.2 percent of individuals score higher than 110 on an IQ test.

It also means that a randomly selected individual has a probability of 0.252 of scoring higher than 110.

The Normal Distribution

Sometimes we are interested in the probability that an observation from a normal population is **between** two given values.

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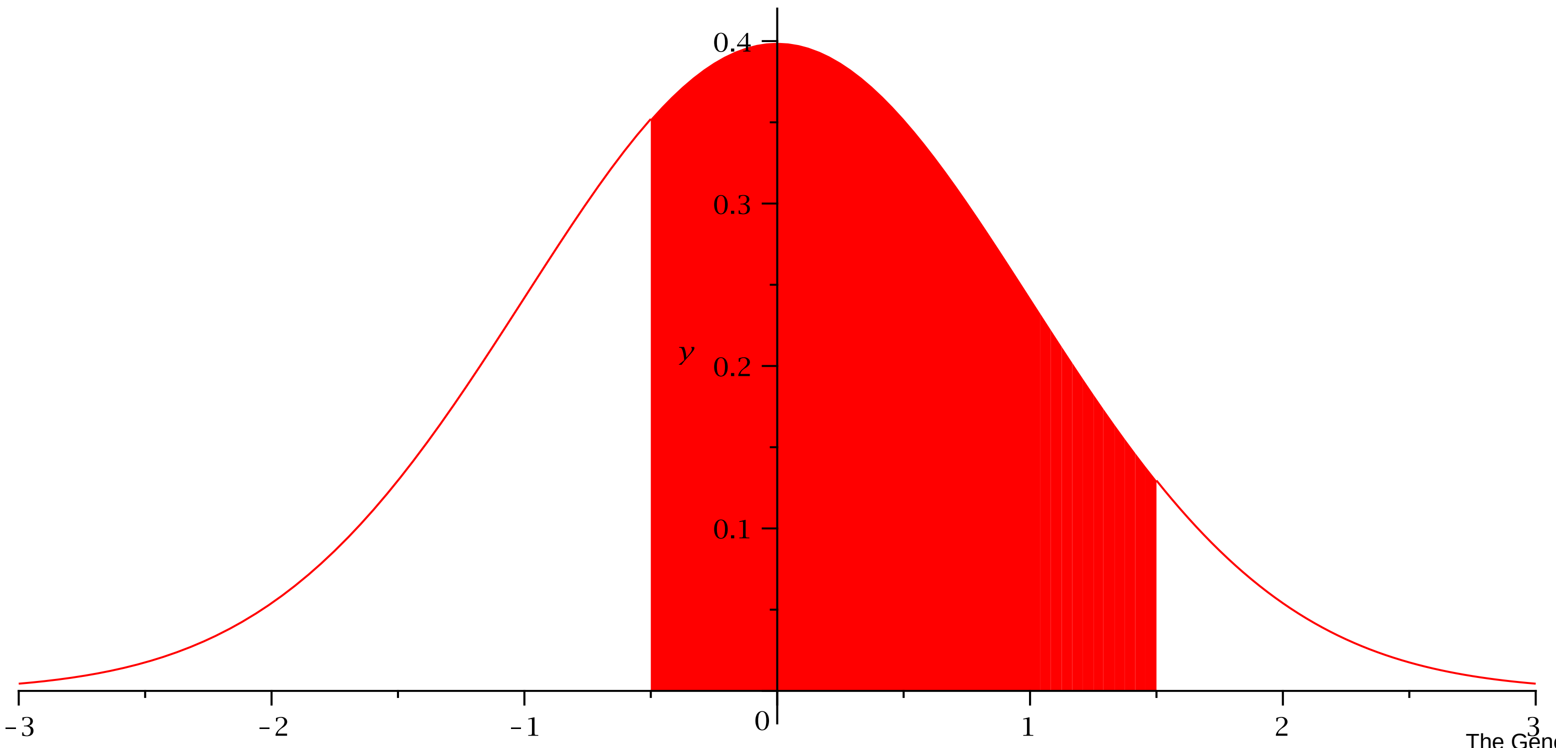
Sometimes we are interested in the probability that an observation from a normal population is **between** two given values.

The area **between** a and b is given by **=NORMSDIST(b,mean,stdev,TRUE)-NORMSDIST(a,mean,stdev,TRUE)**



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IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores between 100 and 110 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)-NORMSDIST(100,100,15,TRUE)**

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores between 100 and 110 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)-NORMSDIST(100,100,15,TRUE)**

The result is 0.248

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores between 100 and 110 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)-NORMSDIST(100,100,15,TRUE)**

The result is 0.248

This means that 24.8 percent of individuals score higher between 100 and 110 on an IQ test.

Example: Normal Distribution

IQ tests are designed so that the mean of the scores is 100 and the standard deviation is 15.

Find the probability that a randomly selected individual scores between 100 and 110 on an IQ test.

Enter **=NORMSDIST(110,100,15,TRUE)-NORMSDIST(100,100,15,TRUE)**

The result is 0.248

This means that 24.8 percent of individuals score higher between 100 and 110 on an IQ test.

It also means that a randomly selected individual has a probability of 0.248 of scoring between 100 and 110.

Example: Normal Distribution

Find the probability that in individual from a normal population with mean 200 and standard deviation 50 lies between 180 and 220.

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Find the probability that in individual from a normal population with mean 200 and standard deviation 50 lies between 180 and 220.

Enter **=NORMSDIST(220,200,50,TRUE)-NORMSDIST(180,200,50,TRUE)**

Example: Normal Distribution

Find the probability that in individual from a normal population with mean 200 and standard deviation 50 lies between 180 and 220.

Enter **=NORMSDIST(220,200,50,TRUE)-NORMSDIST(180,200,50,TRUE)**

The result is 0.311

Example: Normal Distribution

Find the probability that in individual from a normal population with mean 200 and standard deviation 50 lies between 180 and 220.

Enter **=NORMSDIST(220,200,50,TRUE)-NORMSDIST(180,200,50,TRUE)**

The result is 0.311

This means that 31.1 percent of individuals lie between 180 and 220.

Example: Normal Distribution

Find the probability that in individual from a normal population with mean 200 and standard deviation 50 lies between 180 and 220.

Enter **=NORMSDIST(220,200,50,TRUE)-NORMSDIST(180,200,50,TRUE)**

The result is 0.311

This means that 31.1 percent of individuals lie between 180 and 220.

It also means that a randomly selected individual has a probability of 0.311 of scoring between 180 and 220.

Example: Normal Distribution

SAT scores are distributed normally with mean 500 and standard deviation 100.

Find the proportion of the population that scores between 450 and 600.

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SAT scores are distributed normally with mean 500 and standard deviation 100.

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The result is 0.533

Example: Normal Distribution

SAT scores are distributed normally with mean 500 and standard deviation 100.

Find the proportion of the population that scores between 450 and 600. Enter **=NORMSDIST(600,500,100,TRUE)-NORMSDIST(450,500,100,TRUE)**

The result is 0.533

This means that 53.3 percent of the population scores between 450 and 600.

Example: Normal Distribution

SAT scores are distributed normally with mean 500 and standard deviation 100.

Find the proportion of the population that scores between 450 and 600. Enter **=NORMSDIST(600,500,100,TRUE)-NORMSDIST(450,500,100,TRUE)**

The result is 0.533

This means that 53.3 percent of the population scores between 450 and 600.

It also means that an individual selected randomly has a probability of 0.533 of scoring between 450 and 600.

The Normal Distribution

Finally, we may be interested in the probability that an observation from a normal population is **outside** the interval between two given values.

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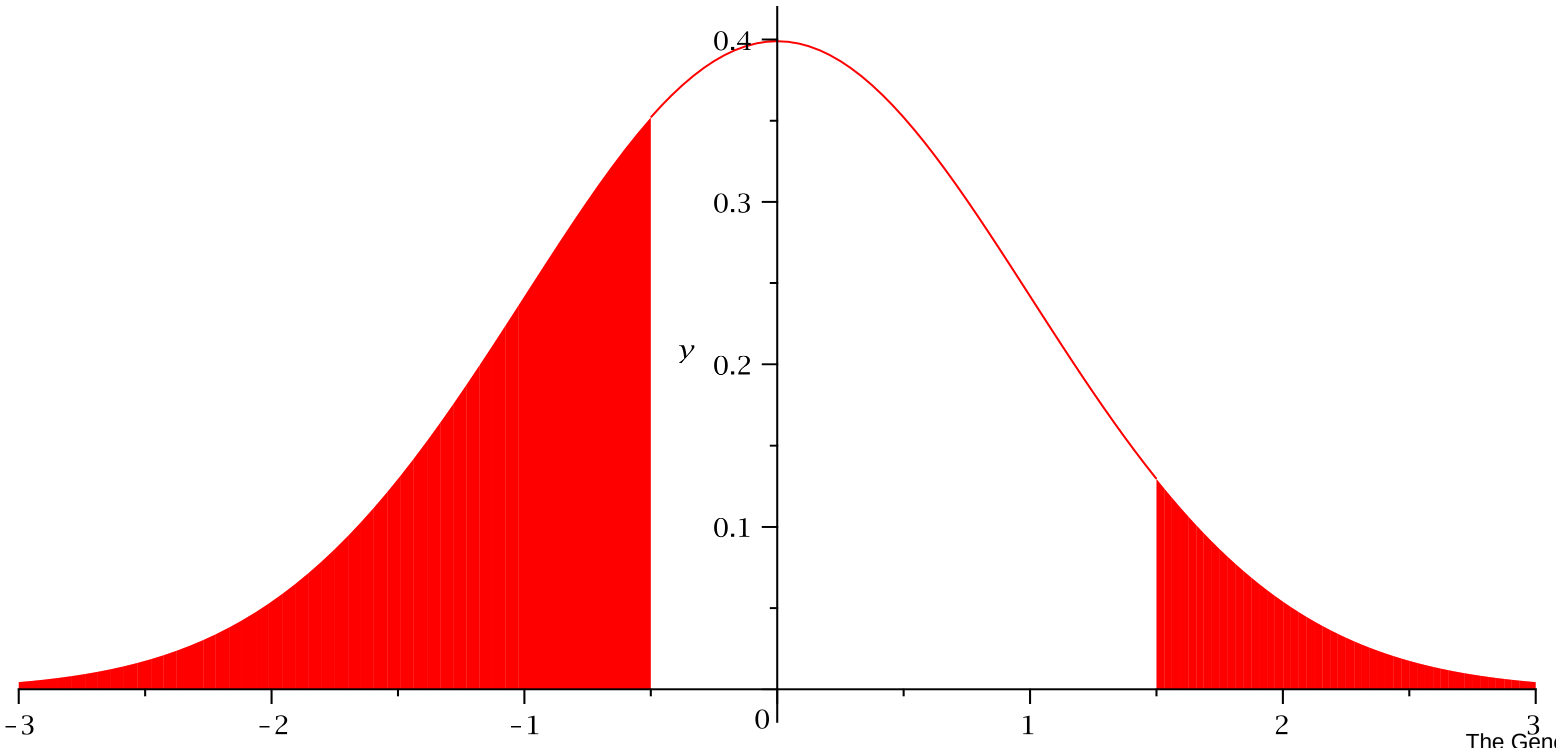
Finally, we may be interested in the probability that an observation from a normal population is **outside** the interval between two given values.

The area **outside** the interval between a and b is given by **=1-NORMDIST(b,mean,stdev,TRUE)+NORMSDIST(a,mean,stde**



Example: Standard Normal Distributio

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Find the proportion of the population that scores less than 450 or greater than 600 on the SAT (mean=500, stdev=100).

Enter **=1-NORMDIST(600,500,100,TRUE)+NORMSDIST(450,500,100,T**

The result is 0.467

Example: Standard Normal Distributio

Find the proportion of the population that scores less than 450 or greater than 600 on the SAT (mean=500, stdev=100).

Enter **=1-NORMDIST(600,500,100,TRUE)+NORMSDIST(450,500,100,T**

The result is 0.467

This means that 46.7 percent of the population scores less than 450 or greater than 600.

Example: Standard Normal Distributio

Find the proportion of the population that scores less than 450 or greater than 600 on the SAT (mean=500, stdev=100).

Enter **=1-NORMDIST(600,500,100,TRUE)+NORMSDIST(450,500,100,T**

The result is 0.467

This means that 46.7 percent of the population scores less than 450 or greater than 600.

It also means that an individual selected randomly has a probability of 0.467 of scoring less than 450 or greater than 600.

Percentiles

Now consider the opposite problem. Suppose we want to find the value x with the property that a given proportion of a standard normal population is less than x.

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This is the same as finding *percentiles* of the standard normal distribution.

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The function **NORMSINV(p)** takes a proportion p, and returns the value x with the property that p is the proportion of a standard normal population that is less than x.

Percentiles

Example: Find the value x with the the property that 74 percent of SAT scores are less than x

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Solution: Enter **=NORMINV(0.74,500,100)**

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Example: Find the value x with the the property that 74 percent of SAT scores are less than x

Solution: Enter **=NORMINV(0.74,500,100)**

The result is 564.33, which means that 74 percent of SAT scores are less than 564.

Percentiles

Example: Find the value x with the the property that 50 percent of a normal population with mean 75 and standard deviation 5 is less than x

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Solution: Enter **=NORMINV(0.50,75,5))**

The result is 75, which means that 50 percent of the population is less than 75.

Percentiles

Example: Find the value x with the the property that 50 percent of a normal population with mean 75 and standard deviation 5 is less than x

Solution: Enter **=NORMINV(0.50,75,5))**

The result is 75, which means that 50 percent of the population is less than 75.

This agrees with the fact that the normal distribution is symmetric about its mean.

Percentiles

Example: Find the 25^{th} percentile of the distribution of IQ scores (mean=100, stdev=15)

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Solution: Enter **=NORMINV(0.25,100,15)**

The result is 89.88, which means that 25 percent of a standard normal population is less than 90.

Percentiles

Example: Find the 90^{th} percentile of SAT scores (mean=500, stdev=100)

Percentiles

Example: Find the 90^{th} percentile of SAT scores (mean=500, stdev=100)

Solution: Enter **=NORMINV(0.90,500,100)**

Percentiles

Example: Find the 90^{th} percentile of SAT scores (mean=500, stdev=100)

Solution: Enter **=NORMINV(0.90,500,100)**

The result is 628.15, which means that 90 percent of people score less than 629.