The Normal or Bellcurve Distribution

Gene Quinn

The Normal Distribution

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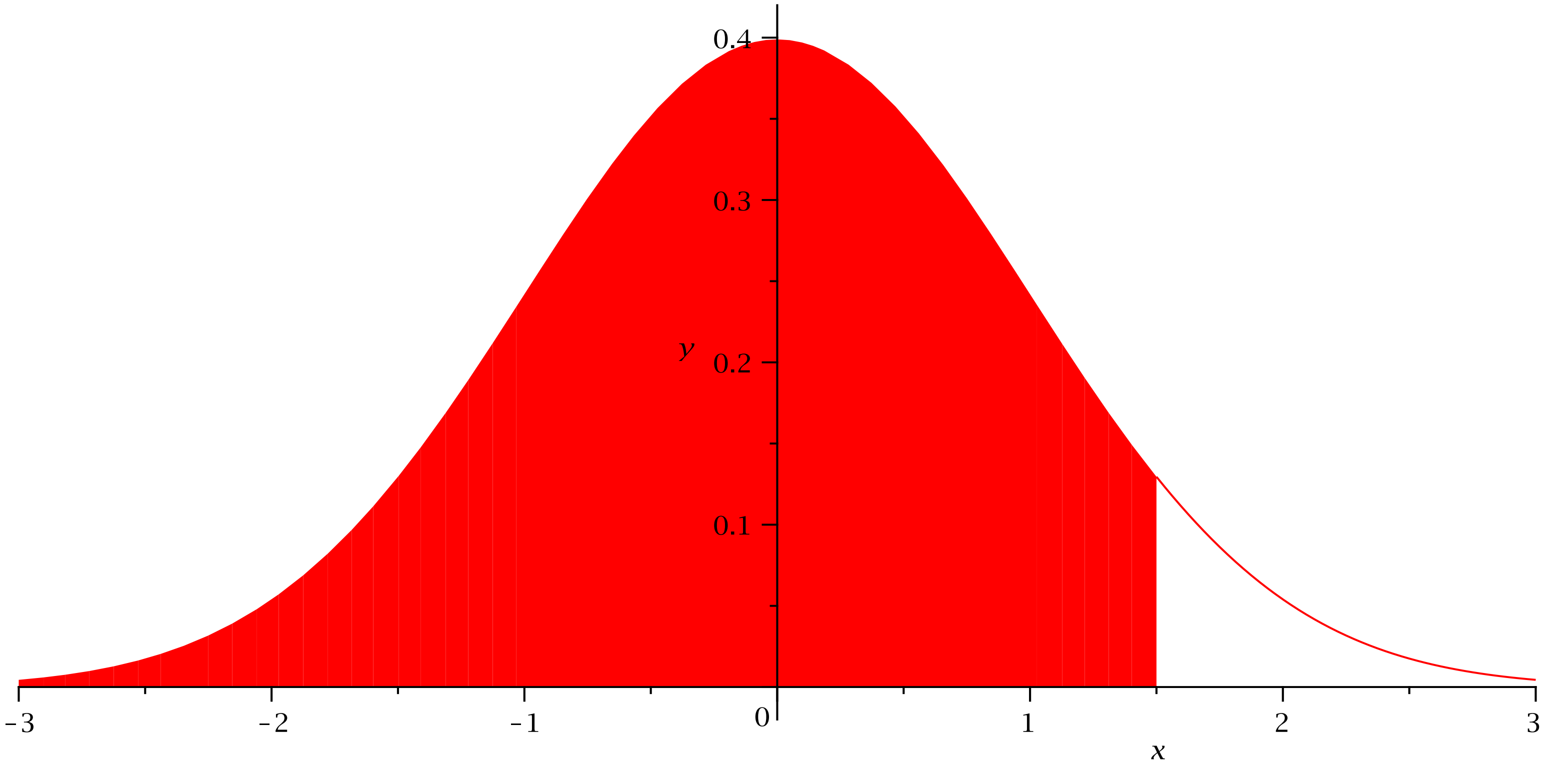

The Normal Distribution

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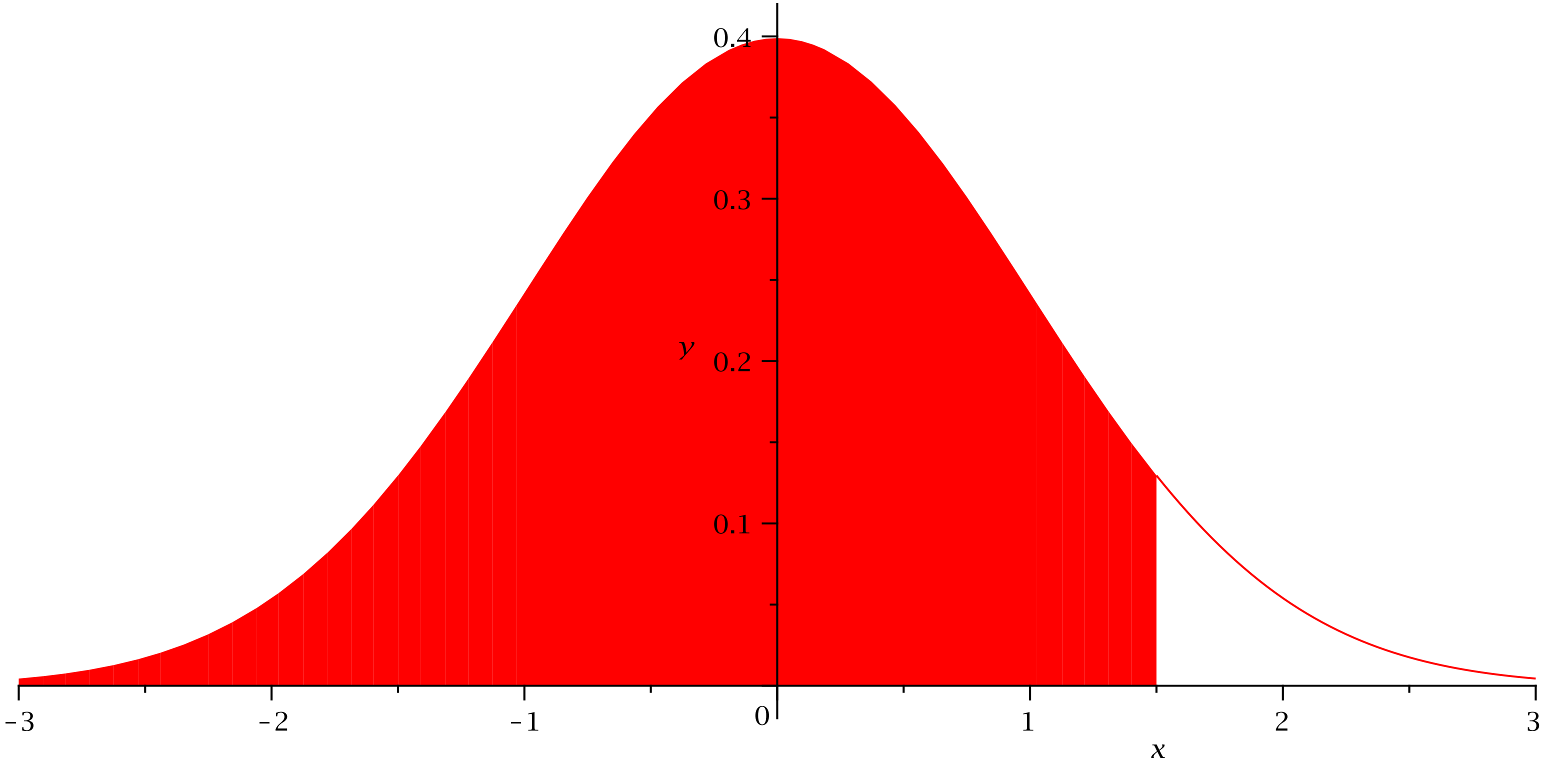

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A bellcurve with mean 0 and standard deviation 1 is called a **standard normal** distribution.

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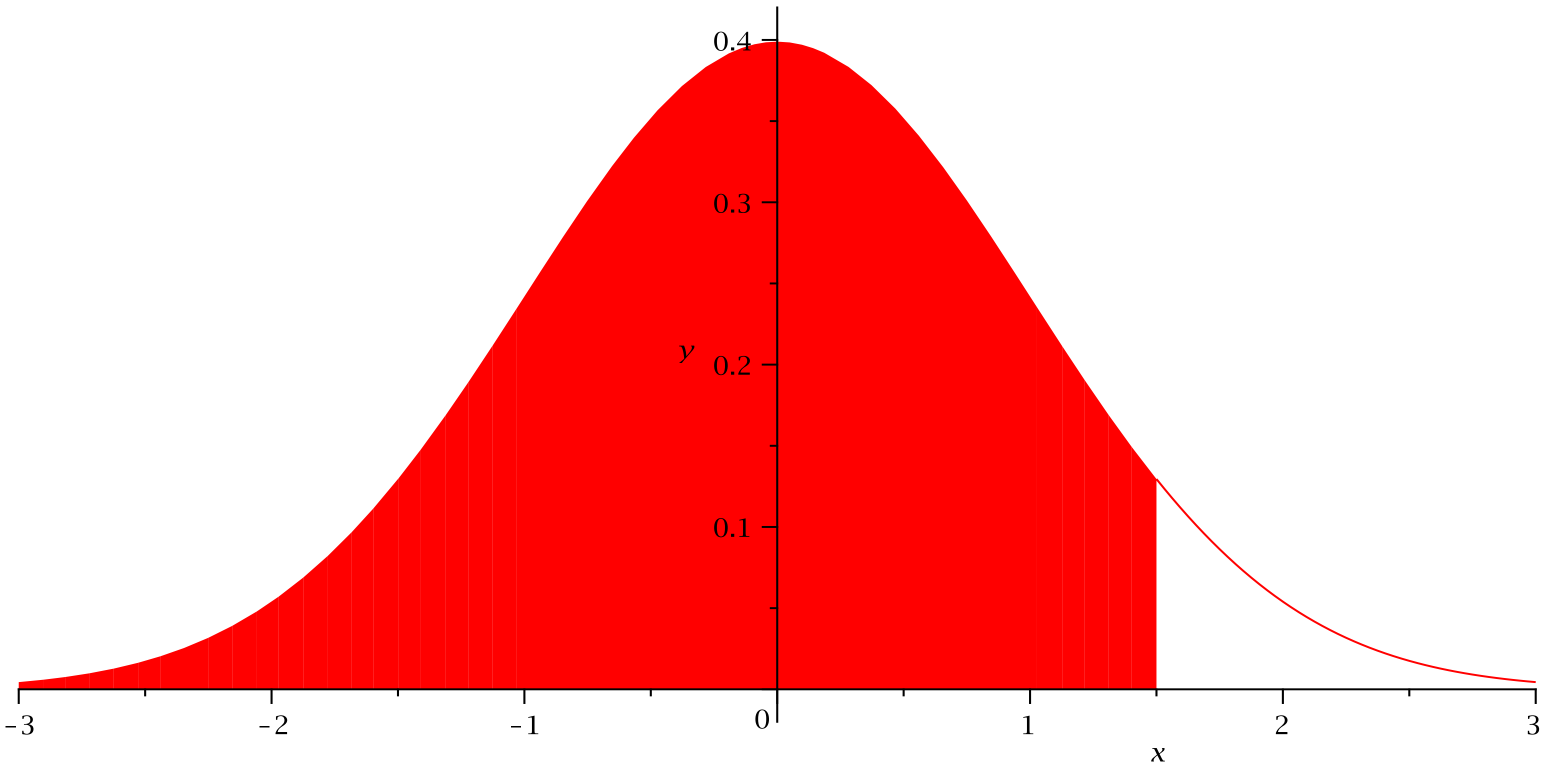

The Normal Distribution

The proportion of the population to the left of a given value is equal to the area under the curve from that point left.

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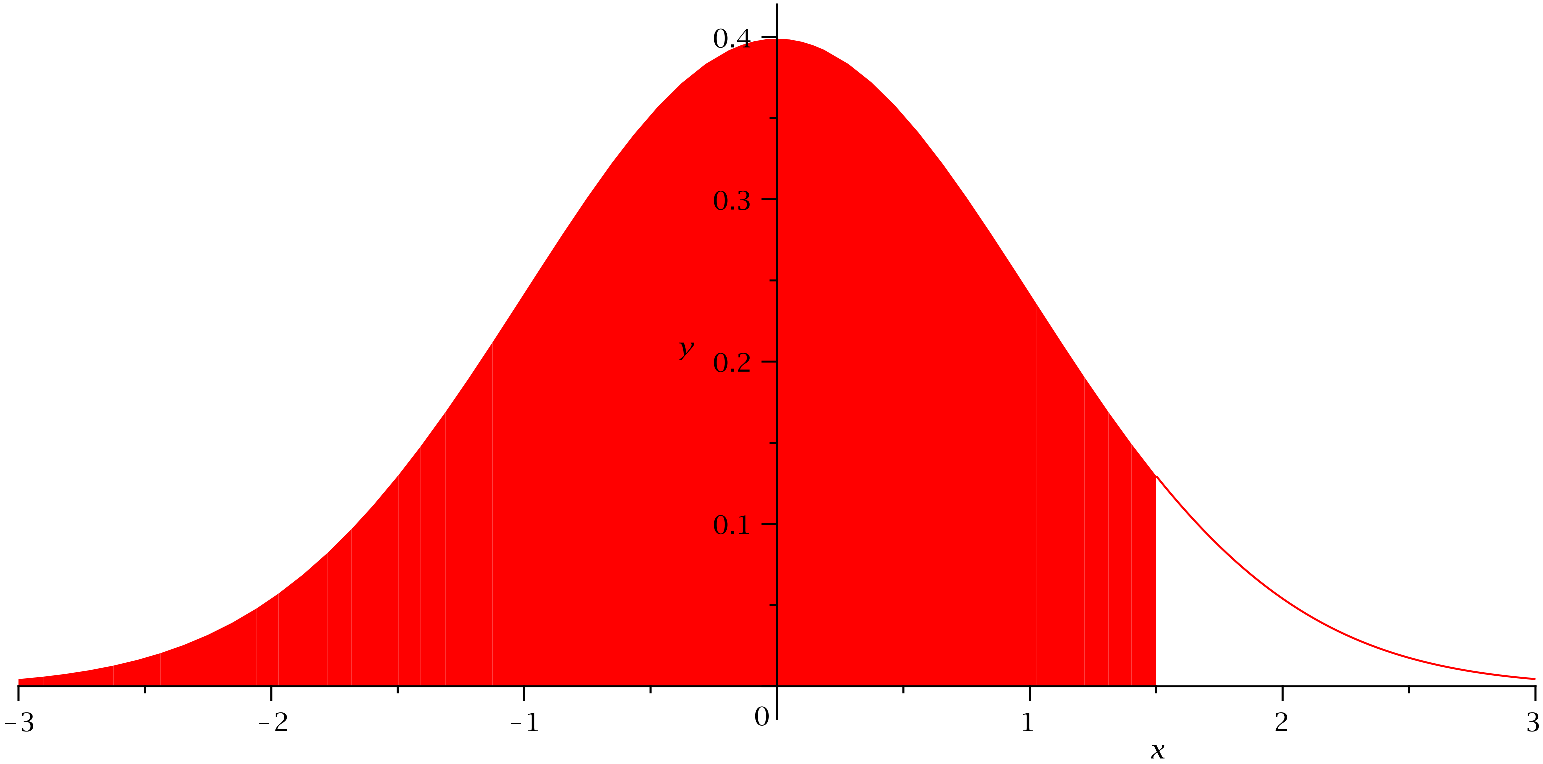

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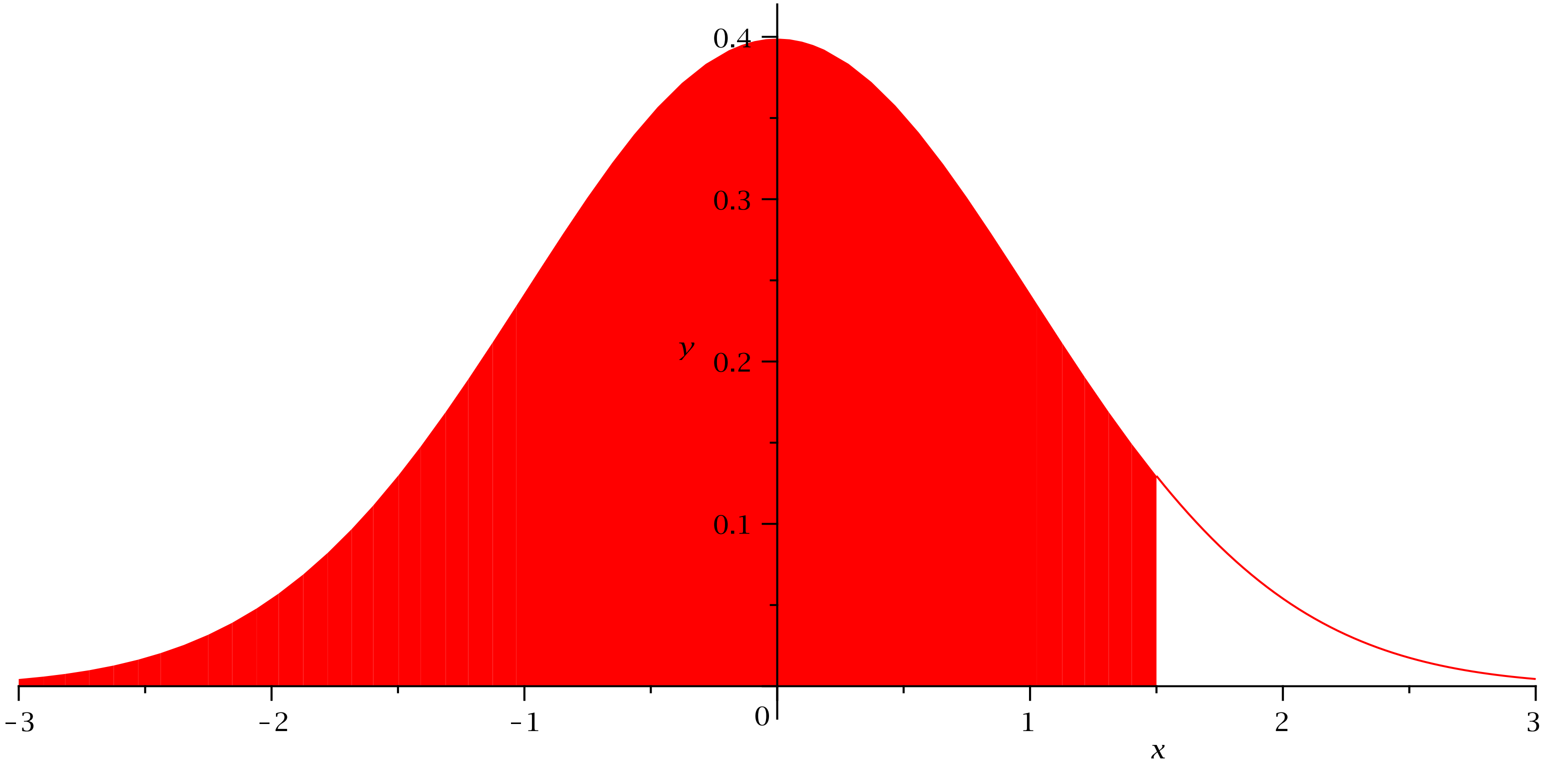

The Normal Distribution

The **NORMSDIST** function takes a single argument, call it z.

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The area to the left of z is given by **=NORMSDIST(z)**

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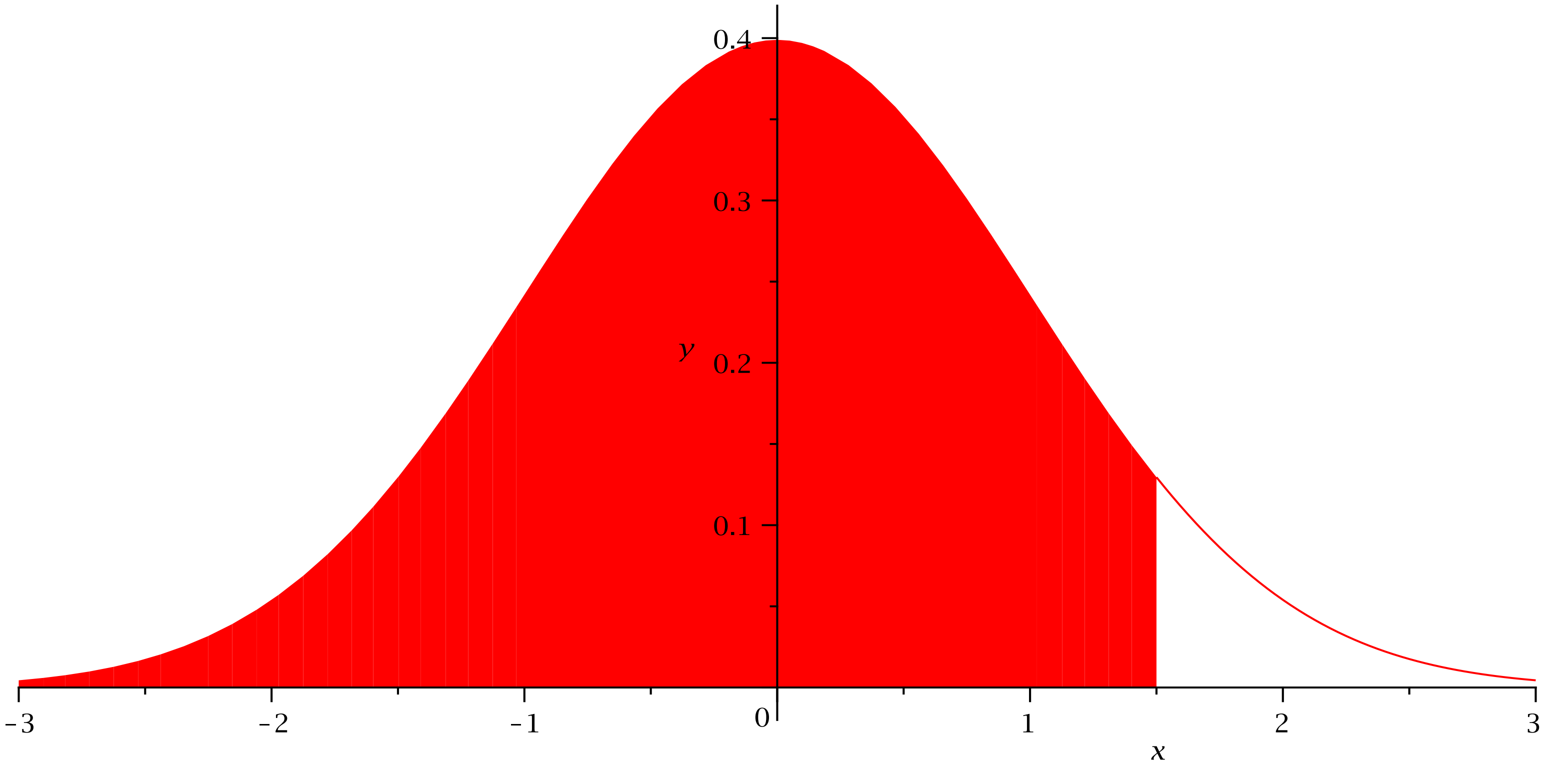

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is less than 1.5.

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It also means that an individual selected randomly from a standard normal population has a probability of 0.933 of being less than 1.5.

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Example: Standard Normal Distributio

Find the proportion of a standard normal population that is less than zero.

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Find the proportion of a standard normal population that is less than zero.

Enter **=NORMSDIST(0.0)**. The result is 0.5

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is less than -2.

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Find the proportion of a standard normal population that is less than -2.

Enter **=NORMSDIST(-2)**. The result is 0.02275

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Find the proportion of a standard normal population that is less than 1.75.

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Find the proportion of a standard normal population that is less than 1.75.

Enter **=NORMSDIST(1.75)**. The result is 0.9599

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Sometimes we are interested in the probability that an observation from a standard normal is **greater than** a given value.

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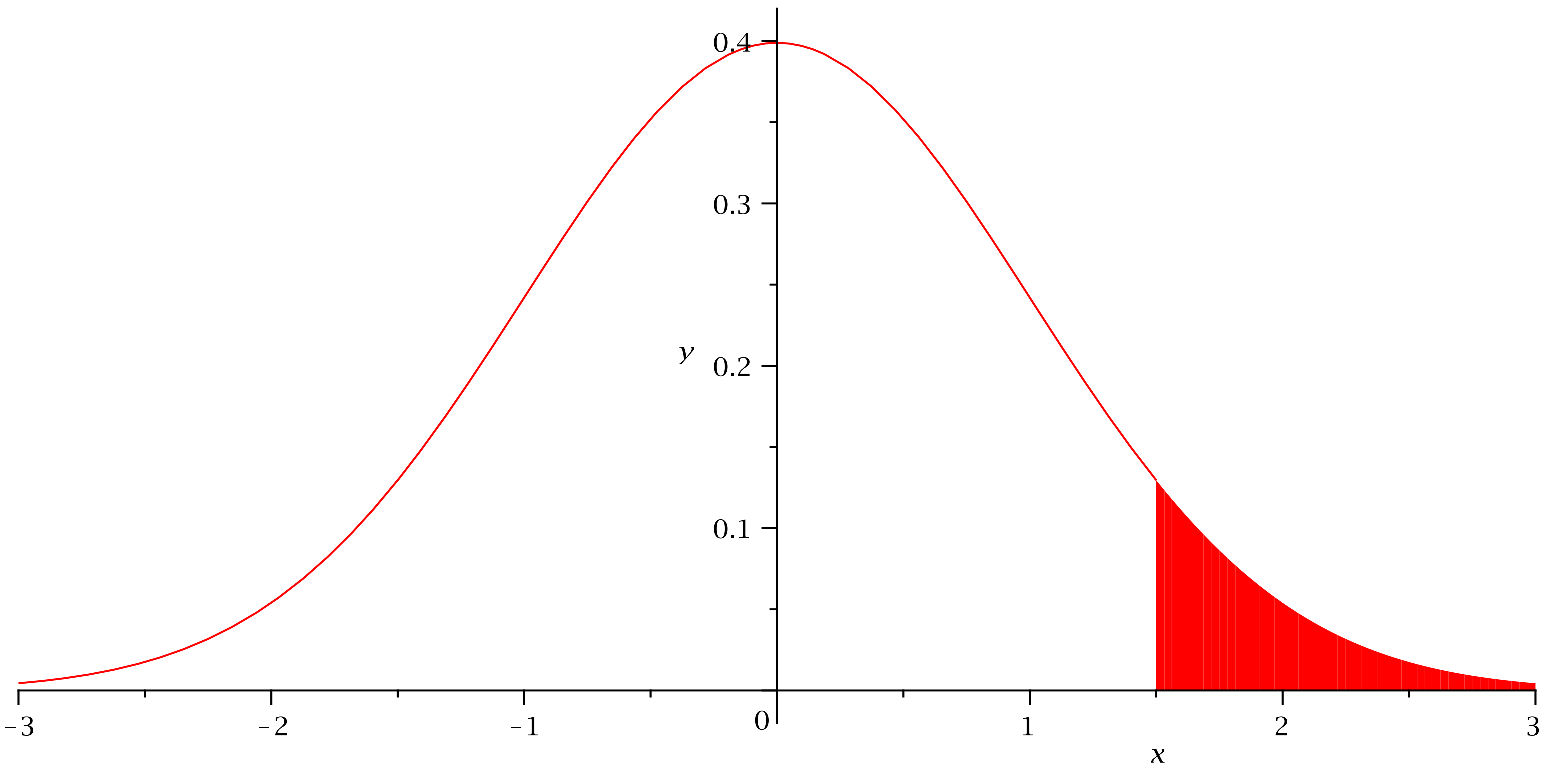
Sometimes we are interested in the probability that an observation from a standard normal is **greater than** a given value.

The area to the **right** of x is given by **=1-NORMSDIST(x)**



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This means that 6.68 percent of a standard normal population has a value greater than 1.5

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The result is 0.0668

This means that 6.68 percent of a standard normal population has a value greater than 1.5

It also means that an individual selected randomly from a standard normal population has a probability of 0.0668 of being greater than 1.5.

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This means that 61.8 percent of a standard normal population has a value greater than -0.3

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The result is 0.618

This means that 61.8 percent of a standard normal population has a value greater than -0.3

It also means that an individual selected randomly from a standard normal population has a probability of 0.618 of being greater than -0.3.

The Normal Distribution

Sometimes we are interested in the probability that an observation from a standard normal is **between** two given values.

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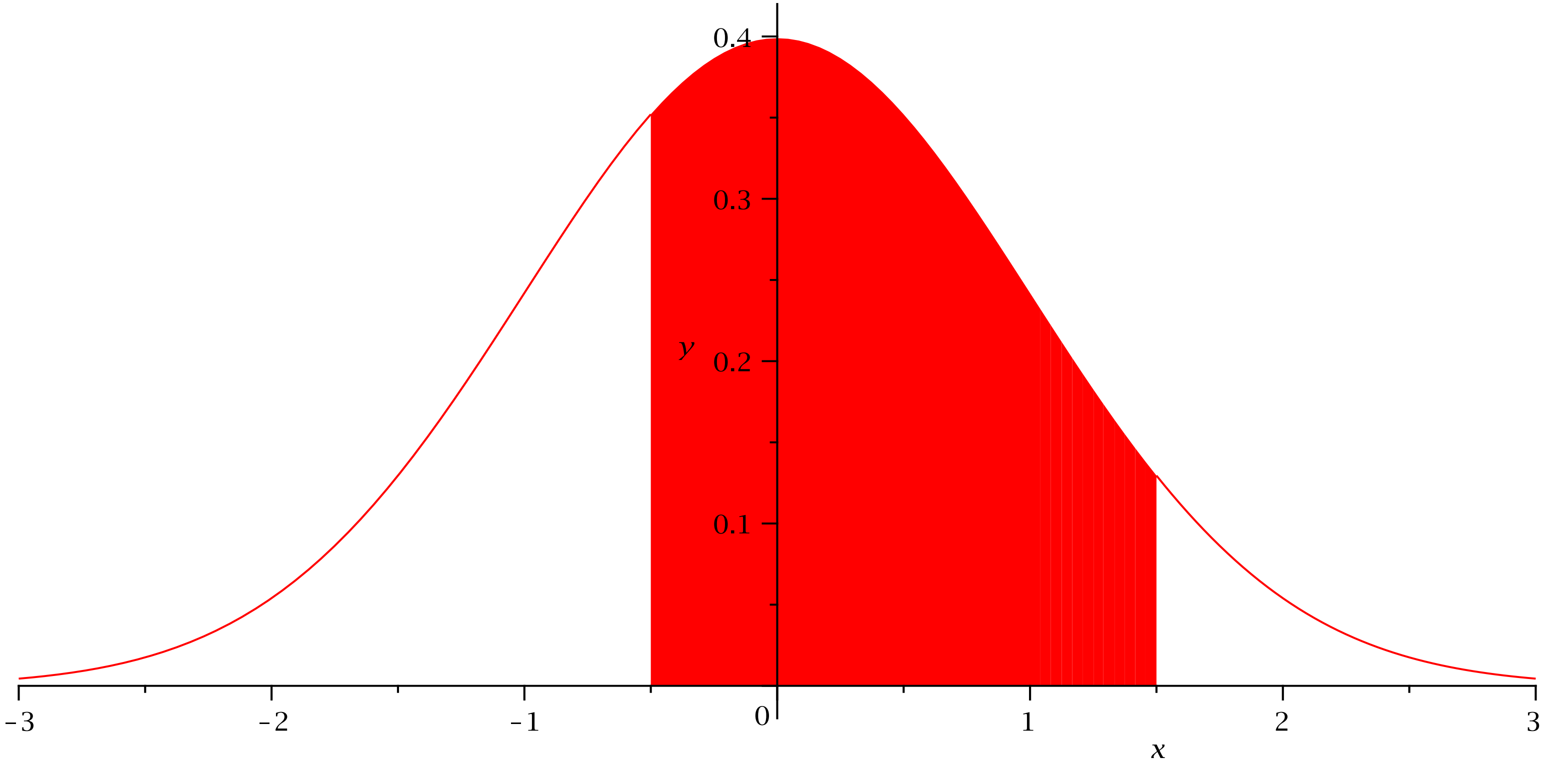
Sometimes we are interested in the probability that an observation from a standard normal is **between** two given values.

The area **between** a and b is given by
=NORMSDIST(b)-NORMSDIST(a)



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This means that 13.6 percent of a standard normal population has a value between 1 and 2.

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Find the proportion of a standard normal population that is between 1 and 2.

Enter **=NORMSDIST(2)-NORMSDIST(1)**

The result is 0.136

This means that 13.6 percent of a standard normal population has a value between 1 and 2.

It also means that an individual selected randomly from a standard normal population has a probability of 0.136 of being between 1 and 2.

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is between -1 and 1.

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Find the proportion of a standard normal population that is between -1 and 1.

Enter **=NORMSDIST(2)-NORMSDIST(1)**

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The result is 0.683

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is between -1 and 1.

Enter **=NORMSDIST(2)-NORMSDIST(1)**

The result is 0.683

This means that 68.3 percent of a standard normal population has a value between -1 and 1.

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is between -1 and 1.

Enter **=NORMSDIST(2)-NORMSDIST(1)**

The result is 0.683

This means that 68.3 percent of a standard normal population has a value between -1 and 1.

It also means that an individual selected randomly from a standard normal population has a probability of 0.683 of being between -1 and 1.

The Normal Distribution

Finally, we may be interested in the probability that an observation from a standard normal is **outside** the interval between two given values.

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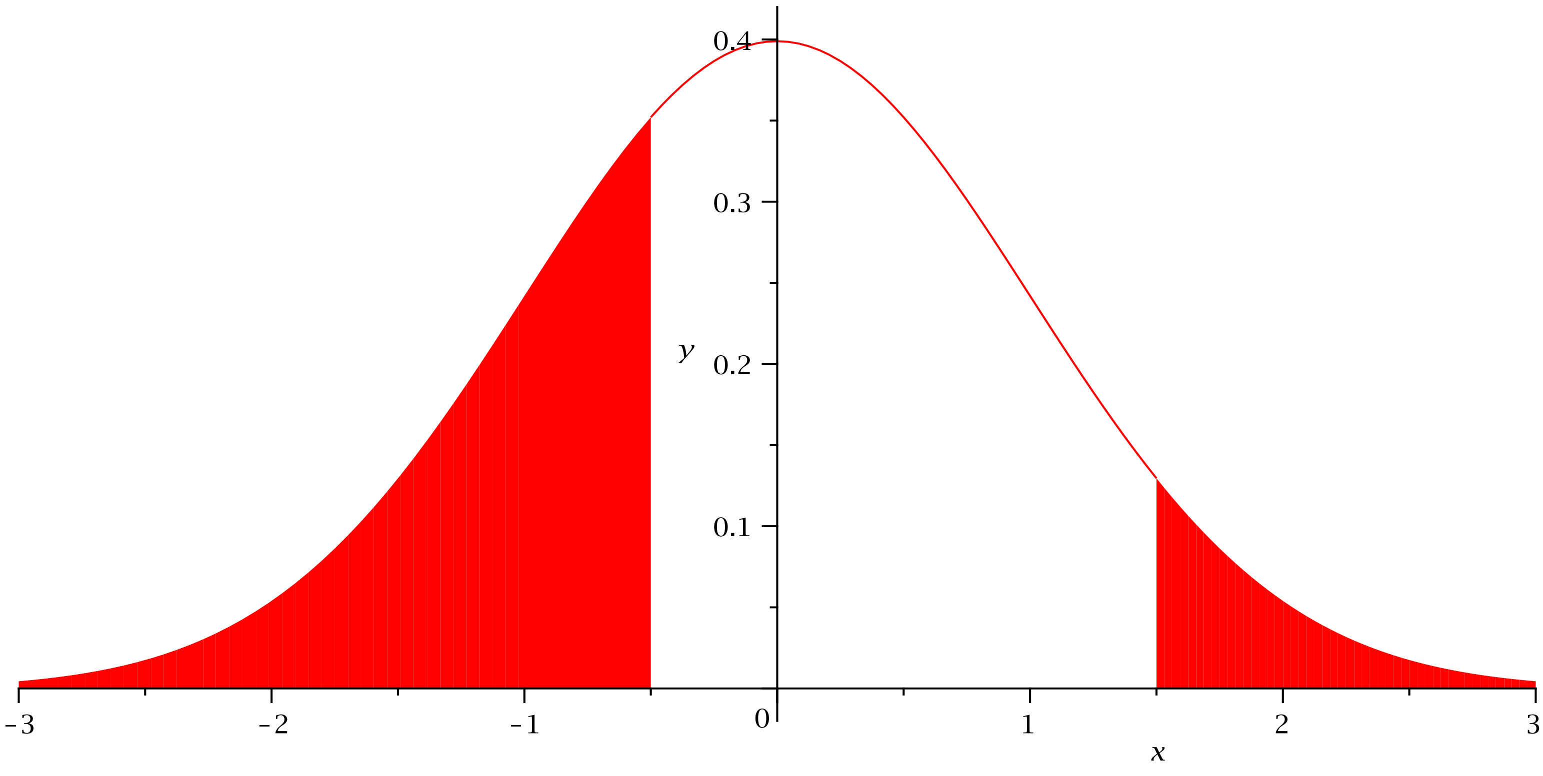
Finally, we may be interested in the probability that an observation from a standard normal is **outside** the interval between two given values.

The area **outside** the interval between a and b is given by
=1-NORMSDIST(b)+NORMSDIST(a)



Example: Standard Normal Distributio

Find the proportion of a standard normal population that is less than 1 or greater than 2.

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The result is 0.864

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The result is 0.864

This means that 86.4 percent of a standard normal population has a value less than 1 or greater than 2.

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is less than 1 or greater than 2.

Enter **=1-NORMSDIST(2)+NORMSDIST(1)**

The result is 0.864

This means that 86.4 percent of a standard normal population has a value less than 1 or greater than 2.

It also means that an individual selected randomly from a standard normal population has a probability of 0.846 of being less than 1 or greater than 2.

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is less than -1 or greater than 1.

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The result is 0.317

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Find the proportion of a standard normal population that is less than -1 or greater than 1.

Enter **=1-NORMSDIST(2)+NORMSDIST(1)**

The result is 0.317

This means that 31.7 percent of a standard normal population has a value less than -1 or greater than 1.

Example: Standard Normal Distributio

Find the proportion of a standard normal population that is less than -1 or greater than 1.

Enter **=1-NORMSDIST(2)+NORMSDIST(1)**

The result is 0.317

This means that 31.7 percent of a standard normal population has a value less than -1 or greater than 1.

It also means that an individual selected randomly from a standard normal population has a probability of 0.317 of being less than -1 or greater than 1.

Percentiles

Now consider the opposite problem. Suppose we want to find the value x with the property that a given proportion of a standard normal population is less than x.

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This is the same as finding *percentiles* of the standard normal distribution.

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This is the same as finding *percentiles* of the standard normal distribution.

The function **NORMSINV(p)** takes a proportion p, and returns the value x with the property that p is the proportion of a standard normal population that is less than x.

Percentiles

Example: Find the value x with the the property that 74 percent of a standard normal population is less than x

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Solution: Enter **=NORMSINV(0.72)**

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Solution: Enter **=NORMSINV(0.72)**

The result is 0.583, which means that 72 percent of a standard normal population is less than 0.583.

Percentiles

Example: Find the value x with the the property that 50 percent of a standard normal population is less than x

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Solution: Enter **=NORMSINV(0.50)**

Percentiles

Example: Find the value x with the the property that 50 percent of a standard normal population is less than x

Solution: Enter **=NORMSINV(0.50)**

The result is 0.00, which means that 50 percent of a standard normal population is less than zero.

Percentiles

Example: Find the value x with the the property that 50 percent of a standard normal population is less than x

Solution: Enter **=NORMSINV(0.50)**

The result is 0.00, which means that 50 percent of a standard normal population is less than zero.

This agrees with the fact that the standard normal distribution is symmetric about its mean, zero.

Percentiles

Example: Find the 25^{th} percentile of the standard normal distribution.

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Solution: Enter **=NORMSINV(0.25)**

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The result is -0.674, which means that 25 percent of a standard normal population is less than -0.674.

Percentiles

Example: Find the 90^{th} percentile of the standard normal distribution.

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Percentiles

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The result is 1.282, which means that 90 percent of a standard normal population is less than 1.282.