

Experiments, Outcomes, and Events

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As is often the case, these odd situations are very unlikely and for practical purposes can be ignored.

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The sample space of the experiment of spinning a roulette wheel and dropping the ball in has 38 elements:

00,0,1,2,3,...,34,35,36

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In the roulette experiment, 0 is a simple event, while "even" is not a simple event.

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These rules are from Kolmogorov's axioms, which are the starting point of modern probability theory.

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In this setting, "unusual" is synonymous with "statistically significant"

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If we roll a balanced die many times, the law of large numbers says the proportion of fours will approach $1/6$, the probability of a four on a single roll.
