The Coin Toss Experiment

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We also assume these are equally likely, that is, we are using a fair coin.

Because the probabilities of all outcomes have to add to one, the probabilities of heads and tails must both be $1/2$.

The Law of Large Numbers

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The greater the number of tosses, the closer the proportion gets to $1/2$.

As we will see, if we repeat the coin toss experiment 1,000 times, we expect the number of heads to be between 476 and 524 more than 99% of the time.

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We get an idea of the probability of these events by using the computer to simulate a large number of repetitions of this experiment, and observing the proportions of experiments that result in zero, one, or two heads.

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When we perform the simulation, we should find that we get a single heads outcome about twice as often as we get zero or two heads.

By the empirical approach, we suspect that the probability of getting one heads is twice the probability of getting zero or two heads.

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The classical method says that if an experiment has n equally likely outcomes, and the number of outcomes for which we say event E has occurred is m, the the probability of the event E is:

$$P(E) = \frac{\text{number of ways event } E \text{ can occur}}{\text{number of possible outcomes}} = \frac{m}{n}$$

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Each toss has two outcomes, heads or tails, so the experiment has the following *four* outcomes:

First Toss	Second Toss
H	H
H	T
T	H
T	T

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Each toss has two outcomes, heads or tails, so the experiment has the following *four* outcomes:

First Toss	Second Toss
H	H
H	T
T	H
T	T

If we assume each of the four outcomes are equally likely, each must have probability $1/4$.

Two Coin Tosses

If we consider the total number of heads obtained, there are three possibilities:

First Toss	Second Toss	Number of Heads	Probability
H	H	2	1/4
H	T	1	1/4
T	H	1	1/4
T	T	0	1/4

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H	T	1	1/4
T	H	1	1/4
T	T	0	1/4

Based on this table, we expect the event "one heads" to have probability $1/2$, while the events "zero heads" and "two heads" have probability $1/4$.

Three Coin Tosses

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This time the experiment has the following *eight* outcomes:

First Toss	Second Toss	Third Toss	Number of Heads
H	H	H	3
H	H	T	2
H	T	H	2
H	T	T	1
T	H	H	2
T	H	T	1
T	T	H	1
T	T	T	0

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Possible values for the total number of heads are $0, 1, 2$ and 3.

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- There is one outcome that produces zero heads.
- There are three outcomes that produces one heads.

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On the other hand, there are three outcomes that produce one heads, and three that produce two.

This means the probability of one heads and the probability of two heads are both $3/8$.