Assignment 3: Conditional Probability and Independence

February 7, 2009

1 Overview

In this assignment we will use a spreadsheet to simulate a rather simple slot machine, from which we will get some empirical estimates of the probabilities of various outcomes.

We will think of a slot machine as a mechanical device containing wheels with pictures painted on them. When the lever is pulled, the wheels are all spun. After a few seconds, they are stopped and one of the pictures appears in a window. There is a window for each wheel, so if the machine has three wheels, a row of three pictures appears.

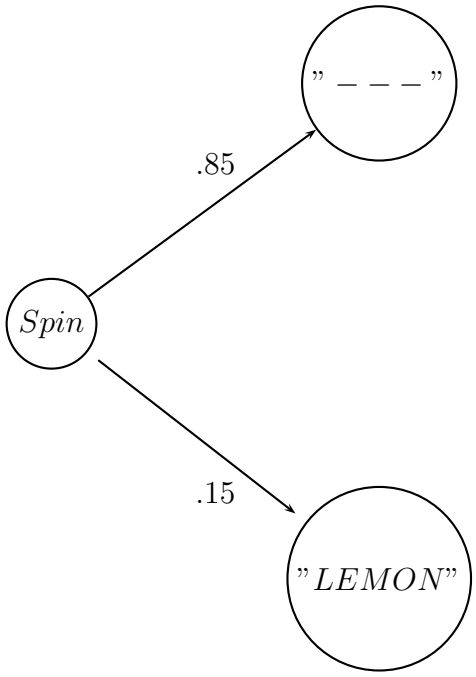
First suppose for simplicity that each wheel in the slot machine has only two pictures:

Picture 1 "- - -"
Picture 2 "LEMON"

When the wheel stops, one of these will appear in the window, although we cannot predict which one.

Usually the pictures do not all have the same probability of appearing. In this case, we will assume that when the wheel is stopped, Picture 1 appears 85% of the time while Picture 2 appears 15% of the time.

We can represent the outcomes of each spin of the wheel by the following diagram:



To implement this with a spreadsheet, we might use the following formula:

$$=\text{IF}(\text{RAND}() < .85,\text{"- - -"},\text{"LEMON"})$$

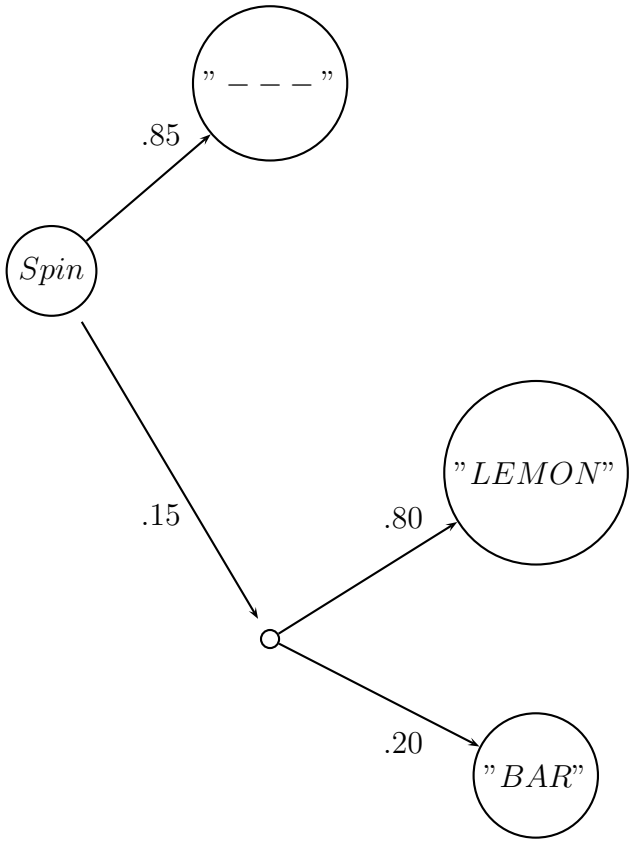
This formula says, in effect, "Draw a number between zero and one, with each number having an equal chance of being chosen. If the number chosen is less than .85, display three dashes. Otherwise, display the word LEMON.

Of course to be realistic we need more than two pictures on each wheel. Suppose we add a third picture,

Picture 1	"- - -"
Picture 2	"LEMON"
Picture 3	"BAR"

As before, suppose that when the wheel is stopped, Picture 1 appears 85% of the time. However, now we will assume that for those times when

Picture 1 does not appear, Picture 2 appears 80% of the time and Picture 3 appears 20% of the time. Now our tree diagram looks like this:



To implement this with a spreadsheet, we might use the following formula. Note that we use two RAND() calls, and two IF functions:

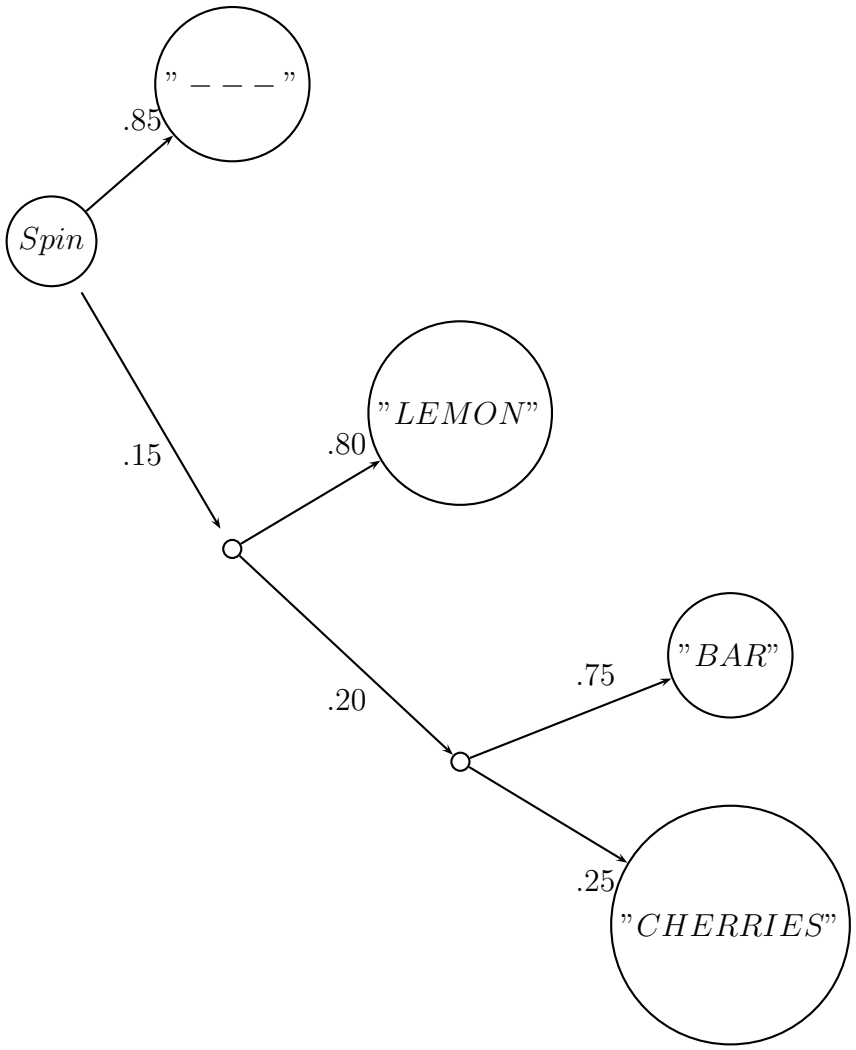
$$=\text{IF}(\text{RAND}() < .85,\text{"- - -"},\text{IF}(\text{RAND}() < .80,\text{"LEMON"},\text{"BAR"}))$$

This formula says, in effect, "Draw a number between zero and one, with each number having an equal chance of being chosen. If the number chosen is less than .85, display three dashes. Otherwise, draw a second number between zero and one. If the second number is less than .80, display the word LEMON. Otherwise, display the word BAR

Finally, suppose we add a fourth picture,

Picture 1	"- - -"
Picture 2	"LEMON"
Picture 3	"BAR"
Picture 3	"CHERRIES"

As before, suppose that when the wheel is stopped, Picture 1 appears 85% of the time. However, now we will assume that for those times when Picture 1 does not appear, Picture 2 appears 80% of the time. When neither Picture 1 nor Picture 2 appears, and Picture 3 appears 75% of the time and Picture 4 appears 25% of the time. Now our tree diagram looks like this:



2 The Assignment

Use the sample spreadsheet as a starting point.

To simulate 1,000 spins of a single wheel, replicate cell F2 into rows 3 through 1001.

To count the number of times each outcome appears, replicate cells G2 (counts dashes), H2 (counts LEMONs), I2 (counts BARs), and J2 (counts CHERRIES) into rows 3 through 1001.

Put a formula to compute the probability of three dashes in cell G1: =SUM(G2:G1001)/1000

Put a formula to compute the (unconditional) probability of LEMON in cell H1: =SUM(H2:H1001)/1000

Put a formula to compute the (unconditional) probability of BAR in cell I1: =SUM(I2:I1001)/1000

Put a formula to compute the (unconditional) probability of CHERRIES in cell J1: =SUM(J2:J1001)/1000

Note that cell B2 should be the probability of three dashes.

Cell B3 is the conditional probability of LEMON given that three dashes DID NOT occur. Would you expect it to match cell H1?

Cell B4 is the conditional probability of BAR given that neither three dashes nor LEMON occured. Does it match cell I1?

There is a simple relationship between the unconditional probabilities in cells H1:J1 and the probabilities supplied in cells B2:B4. Can you discover what it is?

To see what a slot machine with three wheels might produce, replicate cells L2:N2 into a few hundred rows. If the three wheels are independent, what would the probability of getting 3 CHERRIES be?