Sullivan Section 8.1

Gene Quinn

Confidence intervals about a Population Mean

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Example: The **sample mean** $\overline{x}$ is a *point estimate* of the **population mean** μ.

Example: The **sample standard deviation** s is a *point estimate* of the **population standard deviation** σ.

Properties of Point Estimates

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There is no simple answer to this question, because it depends entirely on which criteria you choose to compare different estimates.

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The **expected value** of a statistic is the value that would be obtained if we calculated the statistic for all possible random samples from the population we are studying, and computed the mean of the statistic over all possible samples.

An equivalent statement is that an **unbiased estimate** does not *systematically* underestimate or overestimate the population parameter.

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Consistency is a "large sample" property of an estimator and is a desirable property.

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An estimator that has the smallest possible sample standard deviation among a class of estimators is described simply as **efficient**.

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If anything, quoting a single value creates a false impression of absolute precision: That the population mean is exactly equal to $\overline{x}$

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For a specific estimate $\overline{x}$, the length of this interval will depend on the level of confidence:

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- A narrower interval can be used, but at the expense of lower confidence that it contains μ.

In practice, it is more common to report $\overline{x}$ together with information that allows the computation of an interval with any desired level of confidence.

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The **level of confidence** in a confidence interval is the proportion of time that the interval will contain μ if a large number of samples are taken.

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Somewhat confusingly, a confidence interval associated with a 5% chance of a false positive is referred to as a 95 percent confidence interval.

Computing Confidence Intervals

The formula for computing a confidence interval is as follows:

Lower bound:

$$\text{lower bound} = \overline{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Upper bound:

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The value $z_{\alpha/2}$ represents the value from the z table corresponding to $\alpha/2$, where $1 - \alpha$ is the level of confidence.

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The formula for computing a confidence interval if both the mean and standard deviation have been estimated from the sample uses the t-distribution instead of the z distribution:

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