Sullivan Section 7.2

Gene Quinn

The Standard Normal Distribution

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Because the mean and standard deviation are specified, simply knowing that a random variable has a standard normal distribution tells you everything about that variable.

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- For positive values of x, as x increases the curve approaches zero.
- For negative values of x, as x moves to the left, the curve approaches zero.
- The Empirical rule applies; The standard deviation is $\sigma = 1$

Finding Areas Under the Bell Curve

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For example, we know that the area to the left of zero under the standard normal curve is $1/2$.

This means that 50% of the standard normal population falls below the mean, zero.

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With the advent of computers, the complicated numerical procedures involved can be carried out quickly.

Areas to the Left of z

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For example, to find the area to the left of 1, enter the formula

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Areas to the Left of z

The spreadsheet function **NORMSDIST(z)** returns the area under the standard normal curve to the **left** of z.

For example, to find the area to the left of 1, enter the formula

$$= NORMSDIST(1)$$

The result is 0.841344, which represents the area under the bell curve to the left of 1.

Areas to the Left of z

Example: Find the area under the bell curve to the left of -1

Areas to the Left of z

Example: Find the area under the bell curve to the left of -1

Solution: enter the formula

$$= NORMSDIST(-1)$$

The result is 0.1586, which represents the area under the bell curve to the left of -1.

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Example: Find the area under the standard normal bell curve to the left of zero (we know the answer: 0.5)

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Solution: enter the formula

$$= NORMSDIST(0)$$

The result is 0.5, which represents the area under the bell curve to the left of 0.

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We know the total area under the curve is 1. This means that the area to the **right** of z is:

$$= 1 - NORMSDIST(z)$$

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For example, to find the area to the right of 1, enter the formula

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We know the total area under the curve is 1. This means that the area to the **right** of z is:

$$= 1 - NORMSDIST(z)$$

For example, to find the area to the right of 1, enter the formula

$$= 1 - NORMSDIST(1)$$

The result is 0.1586 (the same as the area to the left of -1).

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Example: Find the area under the standard normal bell curve to the right of of -1

Solution: enter the formula

$$= 1 - NORMSDIST(-1)$$

The result is 0.8413, which represents the area under the bell curve to the right of -1.

Areas Between Two Values

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For the area between z_1 and z_2, enter the formula

$$= NORMSDIST(z_2) - NORMSDIST(z_1)$$

Areas Between Two Values

Example: Find the area under the standard normal bell curve between -1 and 1

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Example: Find the area under the standard normal bell curve between -1 and 1

Solution: enter the formula

$$= NORMSDIST(1) - NORMSDIST(-1)$$

The result is 0.6826, which agrees with the empirical rule.

Areas Between Two Values

Example: Find the area under the standard normal bell curve between -2 and 2

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Example: Find the area under the standard normal bell curve between -2 and 2

Solution: enter the formula

$$= NORMSDIST(2) - NORMSDIST(-2)$$

The result is 0.9545, which also agrees with the empirical rule.

Areas Between Two Values

The values do not have to be equally spaced around the mean.

Example: Find the area under the standard normal bell curve between -1 and 3

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Example: Find the area under the standard normal bell curve between -1 and 3

Solution: enter the formula

$$= NORMSDIST(3) - NORMSDIST(-1)$$

The result is 0.83999, which is the area between -1 and 3.

Areas Outside an Interval

Finally, it is possible to find the area *outside* the interval between two values.

For the area outside the interval from z_1 to z_2, use the formula

$$= NORMSDIST(z_1) + 1 - NORMSDIST(z_2)$$

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Finally, it is possible to find the area *outside* the interval between two values.

For the area outside the interval from z_1 to z_2, use the formula

$$= NORMSDIST(z_1) + 1 - NORMSDIST(z_2)$$

The result sum of the areas to the *left* of z_1 and to the *right* of z_2.

Areas Outside an Interval

Example: Find the area under the standard normal bell curve to the left of -1 and the right of 1

Areas Outside an Interval

Example: Find the area under the standard normal bell curve to the left of -1 and the right of 1

Solution: enter the formula

$$= NORMSDIST(-1) + 1 - NORMSDIST(1)$$

The result is 0.3173, which agrees with the emprical rule.