Sullivan Section 6.2

Gene Quinn

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- Each of the trials is independent of the others. That is, the outcome of one trial has no effect on the other trials.
- The probability of success is the same for each trial

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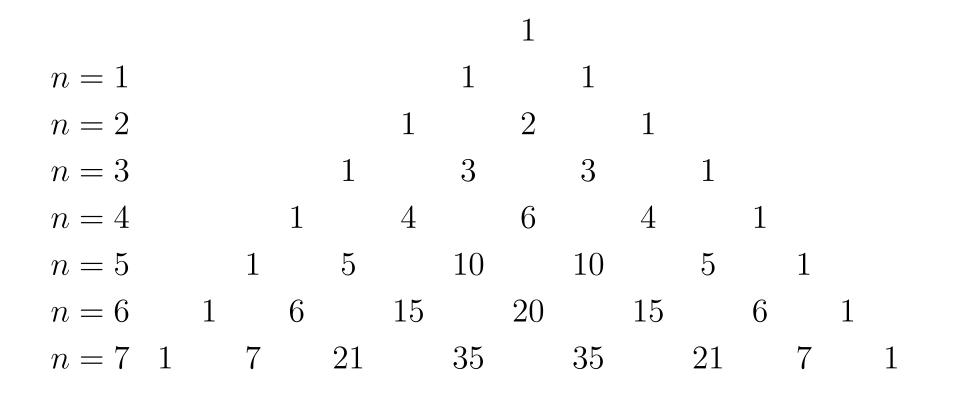
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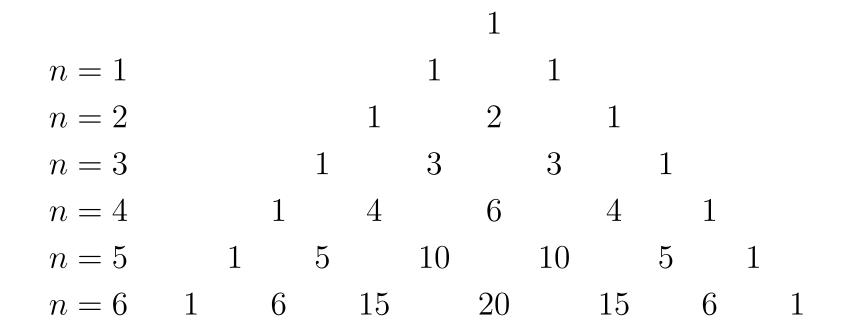
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- The probability of success on each trial is denoted by p.
- The probability of failure on each trial is 1 p.
- The total number of successes in n independent trials is denoted by X.

The following structure known as **Pascal's triangle** is useful for computing binomial probabilities when n is fairly small (n < 10).





The entries in successive rows of Pascal's triangle are the sum of the two closest entries in the previous row.

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The sum of the entries in the row corresponding to n trials is always 2^n .

This represents the number of possible sequences of n letters where each one has to be either S or F.

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- The fourth entry is the number of sequences having 3S's and n - 3 F's

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- The next to last entry is the number of sequences having n 1 S's and 1 F (the next to last entry is always n)
- The last entry is the number of sequences having n S's and 0 F's (the last entry is always 1)

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If we identify "heads" with "success", the experiment corresponds tossing a fair coin n times.

In this case, the probability of obtaining 0, 1, 2, etc. heads in n tosses is the corresponding entry in Pascal's table, divided by the sum of the row (2^n) .

When success and failure are *not* equally likely, we need to use the following modified procedure to calculate the probabilities.

The number of trials n determines which row of Pascal's triangle is used.

Suppose the probability of success on each trial is *p*.

We compute the probabilities associated with each value of X,

where X represents the number of successes in n trials.

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- Continue in this fashion. The $n + 1^{st}$ entry is multiplied by $(1-p)^n$

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First, we need to define another mathematical entity called a **factorial**, which will be designated by a number followed by an exclamation point (!).

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For convenience, we define 0! to be 1.

Definition: The number of **combinations** of n objects taken r at a time is denoted by either

$$_{n}C_{r}$$
 or $\binom{n}{r}$

and is defined to be:

$$\frac{n!}{r!(n-r)!}$$

Computing Binomial Probabilities

Example: Find the number of **combinations** of 4 objects taken 2 at a time.

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By definition,

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1) \cdot (2 \cdot 1)} = \frac{24}{(2)(2)} = 6$$

Computing Binomial Probabilities in G

The general formula for computing the probability of k successes in a binomial experiment with n trials when the probability of success on each trial is p is:

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or, equivalently,

$$P(k \text{ successes}) =_n C_k p^k (1-p)^{n-k}, \quad k = 0, 1, \dots, n$$

Suppose the criteria for a binomial probability experiment are met.

The possible outcomes of the experiment, and the probabilities associated with each outcome are completely determined by two numbers:

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Furthermore, we know that for k = 0, 1, ..., 6, the probability that exactly k successes are obtained is given by the formula:

$$P(X = k) = {}_{6}C_{k} \cdot p^{k}(1-p)^{n-k}$$

In particular, we know that:

• The probability of 0 successes is ${}_{6}C_{0} \cdot (0.6)^{0} (0.4)^{6}$

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The **population standard deviation** σ_X is given by the formula:

$$\sigma_X = \sqrt{n \cdot p \cdot (1-p)}$$

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How large is a "reasonably large" value of n? It depends on p.

A commonly used rule of thumb states that the binomial distribution will be approximately bell shaped provided that

$$n \geq \frac{10}{p \cdot (1-p)}$$

Earlier we found that for a binomial experiment with 100 trials each having a probability of 0.6 of success, the mean and standard deviation were:

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