Hypothesis Testing Procedures

Gene Quinn

Hypothesis Testing (σ Known)

We will assume at first that the population standard deviation σ is known.

The procedure when σ is unknown is very similiar and a separate procedure will be provided for that situation.

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- We are interested in a Two-Tailed test: The alternative hypothesis H_1 states that $\mu \neq \mu_0$, the population mean μ *does not* equal μ_0.

Confidence Interval Method (σ Known)

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