

MA145 Assignment 3 - Confidence Intervals

Name:

1. DOWNLOADING AND EXECUTING THE COMPANION SPREADSHEET

The first step in this assignment is to download and run the accompanying spreadsheet, which is posted on the course website in the Assignment 3 page.

The companion spreadsheet is set up to perform what is known as a *Monte Carlo* experiment. This is a commonly used technique in statistics that uses computer-generated random numbers to simulate a real experiment.

The spreadsheet will generate simulated samples of size $n = 54$ from a normal population with a specified population mean μ and standard deviation σ .

2. OVERVIEW OF SPREADSHEET

Population parameter values and sample size

Population Parameter	Cell	Comments
Population Mean μ	B2	Can be changed. Initial value is zero.
Population Standard Deviation σ	B3	Can be changed. Initial value is one.
Sample size n	B9	Fixed. Value is 54.

Sample statistics (Columns C through L)

Value	Cells	Comments
Sample means $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_{10}$	C6, ..., L6	computed
Sample std deviations s_1, \dots, s_{10}	C7, ..., L7	computed
std errors of means $s_{\bar{x}_1}, \dots, s_{\bar{x}_{10}}$	C10, ..., L10	computed
Sample data values	C25 - C76, ..., L25 - L76	generated

Confidence level and related values

Value	Cell	Comments
confidence level α	B12	Can be changed; Initially 0.05
t -value $t_{\alpha/2}$	B16	computed
z -value $z_{\alpha/2}$	B20	computed

Confidence limits using t -distribution

Value	Cells	Comments
Upper confidence limits	C17 – L17	computed
Lower confidence limits	C18 – L18	computed

Confidence limits using normal distribution

Value	Cells	Comments
Upper confidence limits	C21 – L21	computed
Lower confidence limits	C22 – L22	computed

3. QUESTIONS ON FORMULAS

For each of the following cells, write the formula contained in the cell, the associated mathematical formula (if any), and briefly describe the computations performed in the cell.

Example: Cell B16 contains the formula = $TINV(\$B\$12, \$B\$9)$. This invokes the spreadsheet function $TINV$, with two parameters: the alpha level, and the number of observations (or degrees of freedom). This gives 1/2 the width of the interval centered at the mean of the t distribution that contains $100 \cdot (1 - \alpha)$ percent of the population. The upper limit of the confidence interval for unknown standard deviation (i.e., based on the t distribution) is this value multiplied by the standard error of the sample mean, $\sigma_{\bar{x}}$.

1) Cell C6

2) Cell C7

3) Cell $C10$

4) Cell $C17$

5) Cell $C18$

6) Cell $B20$

7) Cell $C21$

8) Cell $C22$

9) Cell $C25$

4. BINOMIAL DISTRIBUTION (SHEET 2)

Sheet 2 of the spreadsheet contains probability values for a binomial experiment with $n = 10$ trials and probability of success $p = (1 - \alpha)$.

We will consider each sample of 54 values to be a trial of a binomial experiment.

For each of the 10 samples, sheet 1 computes the $100 \cdot (1 - \alpha) - \%$ confidence interval for the population mean μ using the t -distribution.

We will consider the trial a "success" if the confidence interval contains the population mean μ , and a "failure" if it does not.

10) Describe the formula of cell *C11* on sheet 2 and its function.

11) Describe the formula of cell *CD11* on sheet 2 and its function.

12) Count the number of confidence intervals on sheet 1 that contain the population parameter μ (successes). If $\alpha = 0.05$, how many successes do we expect in 10 trials? Is your count consistent with this?

13) Compare the values of $t_{\alpha/2}$ and $z_{\alpha/2}$. What is the difference between the widths of the confidence interval produced from the t distribution (*C17* – *C18*) and that produced using the z -table (*C21* – *C22*)?

5. THE EMPIRICAL RULE

14) Look at cells C25-C76 on sheet 1, the 54 data values in the first trial. How many values are less than -3 or greater than $+3$? From the empirical rule, how many do we expect?

15) Now look at cells less than -2 or greater than $+2$? From the empirical rule, how many do we expect?

16) Now look at cells less than -1 or greater than $+1$? From the empirical rule, how many do we expect?

17) By changing the value in cell *B12* on sheet 1 (the alpha value), determine the width of the interval about the mean, measured in standard deviations, that contains 75% of the population. (First, try changing it to .32 and .05, and compare the $z_{\alpha/2}$ value to the empirical rule).

18) By changing the value in cell *B12* on sheet 1 (the alpha value), determine the width of the interval about the mean, measured in standard deviations, that contains 99.99% of the population. What is the α value in this case?

6. OTHER α VALUES

Change the value in cell *B12* (the alpha level) to 0.10. When you hit the enter key after making the change, the spreadsheet program will generate 10 new samples of 54, and calculate 10 new confidence intervals in rows 17 and 18 of sheet 1.

19) Count the number of confidence intervals on sheet 1 that contain the population parameter μ (successes). If $\alpha = 0.10$, how many successes do we expect in 10 trials? What cell on sheet 2 contains the proportion of trials that we expect to have 10 successes? Is your count consistent with this value?

Change the value in cell *B12* (the alpha level) to 0.20. When you hit the enter key after making the change, the spreadsheet program will generate 10 new samples of 54, and calculate 10 new confidence intervals in rows 17 and 18 of sheet 1.

20) Count the number of confidence intervals on sheet 1 that contain the population parameter μ (successes). If $\alpha = 0.20$, how many successes do we expect in 10 trials? What cell on sheet 2 contains the proportion of trials that we expect to have 10 successes? Is your count consistent with this value?