Integration Strategies

As noted last semester, if a function f is continuous on the *closed* interval $[a, b]$, then the Riemann integral defined by

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

always exists. In this case f is said to be **integrable**.

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Within the class of integrable functions, there is a subset for which we can find an **antiderivative** F,

$$F'(x) = f(x)$$

that can be written as an *elementary function*

Integration Strategies

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This section outlines an organized strategy for finding an antiderivative of a given function.

Note the comment on page 485 under section 4:

There are basically only two methods of integration: substitution and parts

Integration Strategies

Step 1: Try to simplify the integrand

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$$f(x) = \frac{x^2 - 2x + 1}{x - 1}\,dx$$

is much harder to integrate than

$$g(x) = x - 1$$

However, f and g are identical on the domain of f, $\mathbb{R} \setminus \{1\}$

Integration Strategies

Step 2: Try to find a substitution

Try to find a function $u = g(x)$ such that

both $\quad u = g(x) \quad$ and its differential $\quad du = g'(x)\,dx$

occur in the integrand.

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More often than not, substitution in some form is involved in evaluating an integral.

Integration Strategies

Step 3: Try to classify the integrand by form.

Try to recognize the integrand as one of the following:

- Forms for which substitution often works
  - trigonometric functions (substitution - 7.2)
  - rational functions (partial fractions - 7.4)
  - radicals (trigonometric substitition - 7.3)
- Forms for which integration by parts works (7.1)

Integration Strategies

Step 4: Keep trying.

A maxim in the study of problem solving is that one of the big differences between experienced and inexperienced problem solvers is the following:

Inexperienced problem solvers tend to give up much sooner than experienced ones

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These problems, like many in Mathematics or science, often require considerable creativity and ingenuity to solve.

Don't expect to solve every problem in one sitting. If you are stuck, go on to some other activity and return later. Very often a fresh idea occurs after or during such a break. This phenomena is called *incubation*.