Cylindrical Shells Method

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An obvious solution is to just interchange the roles of x and y. The graph of $y = x^2$ is identical to the graph of $x = \sqrt{y}$

So we can just relabel the axes and consider rotating $y = \sqrt{x}$ about the x-axis between $x = 0$ and $x = 1$.

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This works as long as we can convert the function $y = f(x)$ into $x = g(y)$, but this may not be easy.

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The difference in the two volumes is:

$$V_2 - V_1 = \pi h(r_2^2 - r_1^2) \;=\; \pi h(r_2 + r_1)(r_2 - r_1)$$

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This is the volume of a cylindrical shell with height h, average radius r, and thickness Δr.

Question 1

The integral representing the area under the curve $y = x^2$ from $x = 0$ to $x = 1$ revolved around the y-axis is:

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