Integrals of Even and Odd Functions

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Integrals of Even and Odd Functions

Recall that a function f is:

- *even* if $f(-x) = f(x)$
- *odd* if $f(-x) = -f(x)$

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The following theorem is often useful in evaluating integrals of even and odd functions:

Suppose f is continuous on the interval $[-a, a]$.

Then:

$$\int_{-a}^{a} f(x)dx \;=\; 2\int_{0}^{a} f(x)dx \quad \text{if } f \text{ is even}$$

$$\int_{-a}^{a} f(x)dx \;=\; 0 \quad \text{if } f \text{ is odd}$$

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Example: For any odd positive integer k, $f(x) = x^k$ is an odd function, so for any positive real number a and positive odd integer k,

$$\int_{-a}^{a} x^k \, dx = 0$$

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Example: Evaluate:

$$\int_{-\pi}^{\pi} \frac{\sin x}{2 + x^2}\, dx$$

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Example: Evaluate:

$$\int_{-\pi}^{\pi} \frac{\sin x}{2 + x^2}\, dx$$

Substitute $-x$ for x in the above expression, we get

$$\frac{\sin(-x)}{2 + (-x)^2} = \frac{-\sin x}{2 + x^2} = -\frac{\sin x}{2 + x^2}$$

so the function is odd and the integral is zero.

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Example: For any even positive integer k, $f(x) = x^k$ is an even function, so for any positive real number a and positive even integer k,

$$\int_{-a}^{a} x^k \, dx \; = \; 2 \int_{0}^{a} x^k$$