The Fundamental Theorem of Calculus

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Theorem: (Fundamental Theorem of Calculus)

Suppose f is continuous on the (closed) interval $[a, b]$.

Then the function g defined by

$$g(x) \;=\; \int_a^x f(t)\,dt \quad a \leq x \leq b$$

is an antiderivative of f, that is,

$$g'(x) \;=\; f(x) \quad \text{for} \quad a < x < b$$

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Note that in the statement of the theorem, the independent variable x appears as the upper limit of integration.

The variable t in the integrand is merely a placeholder.

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Important!
Note that right hand side is simply the integrand with the placeholder variable t replaced by x.

Do not differentiate the integrand.

The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus is easy to prove using the evaluation theorem if we know an antiderivative.

If F is an antiderivative of F, the evaluation theorem states that

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Differentiating both sides of this expression, we get

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We have now established that

$$\frac{d}{dx}\int_a^x f(t)\,dt \;=\; f(x)$$

Renaming the Evaluation Theorem

We encountered the following result under the name "Evaluation Theorem" in section 5.3

If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx \;=\; F(b) - F(a)$$

where F is any antiderivative of f, that is, $F' = f$.

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The text restates this as the second part of the Fundamental Theorem of Calculus in Section 5.4

FTC Examples

Find

$$\frac{d}{dx}\int_a^x 3t^2\,dt$$

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Solution: Direct application of the FTC gives

$$\frac{d}{dx}\int_a^x 3t^2\,dt \;=\; 3x^2$$

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By the FTC the answer is:

$$g'(x) = \ln x$$