The Net Change Theorem

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Net Change Theorem

This theorem is actually a restatement of the evaluation theorem in a way that relates the integral of the (instantantaneous) rate of change of a quantity over some period to the net change in that quantity:

It states that the integral of the (instantantaneous) rate of change is the net change.

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Theorem: (Net Change Theorem) Suppose $F(t)$ is the amount of some quantity at time t, and $f(t)$ is the derivative of $F(t)$. Then

$$\int_a^b f(t)\,dt \;=\; \int_a^b F'(t)\,dt \;=\; F(b) - F(a)$$

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The net change theorem says that after t seconds, the net change in velocity will be given by:

$$\int_0^t 32\,dx \;=\; 32x\Big|_0^t \;=\; 32t$$

so $v(t) = 32t$.

Net Change Theorem

Now that we have the instantaneous rate of change of position as a function of time,

$$v(t) = s'(t) = 32t$$

we can apply the net change theorem a second time. Now the theorem says that the net change in position after 3 seconds, which is the distance the object has fallen, will be given by

$$\int_0^3 v(t)\,dt = \int_0^3 32t\,dt = 16t^2\Big|_0^3 = 16 \cdot 9 - 16 \cdot 0 = 144 \text{ (feet)}$$

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Example: A valve at the bottom of a tank is opened at time $t = 0$. The volume of water V in a tank changes at the rate $V'(t)$ gallons per hour once the valve is opened. Find the net change in the volume of water V in the first 5 hours after the valve is opened.

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Solution: The instantaneous rate of change of the volume of water is $V'(t)$. The net change in volume is:

$$\int_0^5 V'(t)\,dt \;=\; V(5) - V(0)$$

Without more information on V', this is the best answer we can give.