Indefinite Integrals

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$$\int f(x)\,dx = F(x)$$

The left hand side of this equation,

$$\int f(x)dx$$

is called an **indefinite integral** of f

An indefinite integral and an antiderivative are the same thing.

Definite and Indefinite Integrals

The evaluation theorem (also known as the second statement of the fundamental theorem of calculus) says that if F is an antiderivative of f,

$$\int_a^a f(x)dx = F(a) - F(a) = 0$$

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The evaluation theorem (also known as the second statement of the fundamental theorem of calculus) says that if F is an antiderivative of f,

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Likewise, if $a \leq c \leq b$,

$$\int_a^c f(x)dx + \int_c^b f(x)dx$$

$$= F(c) - F(a) \quad + \quad F(b) - F(c)$$

$$= F(b) - F(a) = \int_a^b f(x)dx$$

Evaluating Definite Integrals

Example:

Find

$$\int_0^3 3x^2 dx$$

Evaluating Definite Integrals

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Find

$$\int_0^3 3x^2 dx$$

$$F(x) = x^3 + C$$

is the general antiderivative of f.

The FTC says

$$\int_0^3 f(x)dx = F(3) - F(0)$$

$$= 3^3 - 0^3 = 27$$

Evaluating Definite Integrals

Example:

Evaluate

$$\int_0^{2\pi} \sin x \, dx$$

Evaluating Definite Integrals

Example:

Evaluate

$$\int_0^{2\pi} \sin x \; dx$$

$$F(x) = -\cos x + C$$

is the general antiderivative of f.

The FTC says

$$\int_0^{2\pi} \sin x dx = F(2\pi) - F(0)$$

$$= -\cos(2\pi) + \cos(0) = -1 + 1 = 0$$