The Evaluation Theorem

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$$\int_a^b f(x)\,dx \;=\; F(b) - F(a)$$

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Later, we will incorporate this theorem into the Fundamental Theorem of Calculus.

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Example: Evaluate

$$\int_0^1 x^2 \, dx$$

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Note that

$$F(x) = \frac{x^3}{3}$$

is an antiderivative of $f(x) = x^2$. The evaluation theorem states that:

$$\int_0^1 x^2\,dx = F(1) - F(0) = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$

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$$\int_0^{\pi} \cos x \, dx$$

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In this case

$$F(x) = \sin x$$

is an antiderivative of $f(x) = \cos x$. The evaluation theorem states that:

$$\int_0^{\pi} \cos \, dx = F(\pi) - F(0) = \sin \pi - \sin 0 = 0 - 0 = 0$$