The Evaluation Theorem

Gene Quinn
The Evaluation Theorem

**Theorem:** If $f$ is continuous on the interval $[a, b]$, then

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\int_{a}^{b} f(x) \, dx = F(b) - F(a)
$$

where $F$ is any antiderivative of $f$. 
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Later, we will incorporate this theorem into the Fundamental Theorem of Calculus.
The Evaluation Theorem

**Example:** Evaluate

\[ \int_{0}^{1} x^2 \, dx \]
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\[ \int_{0}^{1} x^2 \, dx \]

Note that

\[ F(x) = \frac{x^3}{3} \]

is an antiderivative of \( f(x) = x^2 \). The evaluation theorem states that:

\[ \int_{0}^{1} x^2 \, dx = F(1) - F(0) = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} \]
The Evaluation Theorem

**Example:** Evaluate

\[
\int_{0}^{\pi} \cos x \, dx
\]
The Evaluation Theorem

**Example**: Evaluate

\[ \int_{0}^{\pi} \cos x \, dx \]

In this case

\[ F(x) = \sin x \]

is an antiderivative of \( f(x) = \cos x \). The evaluation theorem states that:

\[ \int_{0}^{\pi} \cos dx = F(\pi) - F(0) = \sin \pi - \sin 0 = 0 - 0 = 0 \]