The Midpoint Rule

Gene Quinn

The Midpoint Rule

Recall the special case where we chose x_i^* to be the midpoint of the i^{th} interval and denoted x_i^* by $\overline{x}_i$:

$$x_i^* = \overline{x}_i = \frac{1}{2}\left(x_{i-1} + x_i\right)$$

so that the Riemann sum became

$$\sum_{i=1}^{n} f(\overline{x}_i)\Delta x$$

The Midpoint Rule

Recall the special case where we chose x_i^* to be the midpoint of the i^{th} interval and denoted x_i^* by $\overline{x}_i$:

$$x_i^* = \overline{x}_i = \frac{1}{2}\left(x_{i-1} + x_i\right)$$

so that the Riemann sum became

$$\sum_{i=1}^{n} f(\overline{x}_i)\Delta x$$

As a technique for approximating a definite integral, this is called the **midpoint rule** and is stated as

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f\left(\overline{x}_i\right)\Delta x = \Delta x[f(\overline{x}_1) + \cdots + f(\overline{x}_n)]$$

The Midpoint Rule

Example: Use the midpoint rule with $n = 4$ to evaluate

$$\int_{-2}^{2} |x| \, dx$$

The Midpoint Rule

Example: Use the midpoint rule with $n = 4$ to evaluate

$$\int_{-2}^{2} |x| \, dx$$

Solution: The endpoints of the four subintervals are: $\{-2, -1, 0, 1, 2\}$.

The midpoints of the subintervals are: $\{-1.5, -0.5, 0.5, 1.5\}$

The function values at the midpoints are: $\{1.5, 0.5, 0.5, 1.5\}$, and the approximate area is:

$$(1 \cdot 1.5 + 1 \cdot 0.5 + 1 \cdot 0.5 + 1 \cdot 1.5) \;=\; 4$$

As it turns out, the approximation has the same value as the exact definite integral in this case.