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Second, even if you find an expression, it may be hard to find its limit.

It would help to have some way of determining whether a series converges or not, without having to actually find the limit.

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You want to pick an integrand that is continuous, positive, decreasing, and interpolates the series you are working with on $[0, \infty)$ (or $[m, \infty)$ for some m). That is,

$$f(n) = a_n \quad n = 1, 2, 3, \ldots$$

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If the integral diverges, the series diverges.

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The result is

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad \text{converges if } \; p > 1 \quad \text{and diverges if } p \leq 1$$

The Comparison Test

If

$$\sum a_n \quad \text{and} \quad \sum b_n$$

are series with positive terms, then:

If $a_n \leq b_n$ for all n and $\sum b_n$ is convergent, then $\sum a_n$ is convergent also.

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If $a_n \geq b_n$ for all n and $\sum b_n$ is divergent, then $\sum a_n$ is divergent also.

Question 1

Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$$

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2. The series diverges

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Solution

The series converges because

$$f(x) = \frac{1}{(2x+1)^3}$$

is continuous, positive, and decreasing on $[1, \infty)$ so

$$\int_1^\infty \frac{1}{(2x+1)^3} = \lim_{t\to\infty} \int_1^t \frac{1}{(2x+1)^3}$$

$$= \lim_{t\to\infty} \left[-\frac{1}{4} \frac{1}{(2x+1)^2} \right]_1^t = \frac{1}{36}$$

Question 2

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Solution

The series converges because

$$a_n = \frac{11}{3n^2 + 5n + 2} < \frac{11}{3n^2} = \frac{11}{3}\frac{1}{n^2}$$

which is a p-series with $n = 2$.