1. Exam 1 Inclass Solutions

Problem 1

Evaluate the integral

$$\int \frac{x^2 + x + 4}{x^3 + 4x} dx$$

Solution: Write the integrand as

$$\frac{2x^2 + x + 4}{x(x^2 + 4)}$$

The partial fractions expansion has the form

$$\frac{2x^2 + x + 4}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + c}{x^2 + 4}$$

Multiplying both sides by $x(x^2 + 4)$ we get

$$\begin{aligned} 2x^2 + x + 4 &= A(x^2 + 4) + (Bx + C)(x) \\ 2x^2 + x + 4 &= x^2(A + B) + Cx + 4A \end{aligned}$$

Equating the coefficients of like powers of x leads to the system of equations

$$\begin{array}{rcccccl} x^2 : & A & + & B & & = & 2 \\ x : & & & & C & = & 1 \\ const : & 4A & & & & = & 4 \end{array}$$

From the last two equations, $C = 1$ and $A = 1$. Substituting $A = 1$ into the first equation gives $B = 1$ so the expansion is

$$\frac{2x^2 + x + 4}{x(x^2 + 4)} = \frac{1}{x} + \frac{x + 1}{x^2 + 4} = \frac{1}{x} + \frac{x}{x^2 + 4} + \frac{1}{x^2 + 4}$$

then

$$\begin{aligned} \int \frac{2x^2 + x + 4}{x(x^2 + 4)} dx &= \int \frac{dx}{x} + \int \frac{x}{x^2 + 4} dx + \int \frac{dx}{x^2 + 2^2} \\ &= \ln|x| + \frac{1}{2} \ln|x^2 + 4| + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C \end{aligned}$$

Problem 2

Find the volume obtained by revolving the area under the curve

$$y = x^2 e^x$$

from $x = 0$ to $x = 1$ around the $y - axis$

Solution: This is a standard cylindrical shells problem, so the volume is

$$V = 2\pi \int_0^1 x \cdot x^2 e^x dx \ = \ 2\pi \int_0^1 x^3 e^x$$

The integral can be evaluated with integration by parts. The table is:

$+$	x^3	e^x
$-$	$3x^2$	e^x
$+$	$6x$	e^x
$-$	6	e^x

and the integral is:

$$2\pi \left[e^x(x^3 + 3x^2 + 6x + 6)\right]_0^1 \ = \ 2\pi(16e - 6)$$

Problem 3 Find the volume obtained by revolving the area under the curve

$$y = \cos x$$

from $x = 0$ to $x = \pi$ around the $x - axis$

Solution: The cylindrical disk method gives:

$$V = \pi \int_0^\pi \cos^2 x \ dx$$

Using the identity

$$\cos^2 x \ = \ \frac{1}{2}(1 + \cos x)$$

by substitution we get

$$V = \frac{\pi}{2} \int_0^\pi (1 + \cos 2x) dx \ = \ \frac{\pi}{2} \left[x + \frac{1}{2} \sin 2x\right]_0^\pi \ = \ \frac{\pi^2}{2}$$