MA126 Exam 2 Study Guide

## 1. In General

Some aspect of my personal philosophy on (in-class) exams:

- The purpose of an exam is to provide a reasonably objective measure of your level of understanding of the material, as well as proficiency using the material to solve problems.
- As a rule, anything covered in class, anything in the sections of the text listed on the syllabus, and anything in supplemental material posted on the website is fair game.
- With that said, I choose exam problems with a view towards what I consider the most important aspects of the material, and class lectures emphasize these aspects.
- When considering a particular question for an exam, I examine the material in the text, the list of suggested problems, and the examples covered in class. Everything you will need to solve the problem should be in one of these sources (This does *not* mean there has to be a similar problem).
- Exam questions are not malicious; they are not designed to trip you up. They *are* designed so that you are unlikely to be able to do them if you are not adequately prepared.
- Exam questions will generally *not* be exact replicas of problems you have already done. However, they will always be chosen so that a student who has read the material, comprehends it reasonably well, and is able to solve the suggested problems and classroom examples is likely to be able to solve them.
- The number of topics usually rules out an exam question specific to each one. Rather than leave out topics, I prefer to make use of questions that combine topics.
- The exams will reflect to some degree the cumulative nature of the subject. New material ueually builds on earlier topics. Previous topics not specifically included in the sections covered on the exam may be needed to solve problems related to the new material. The first problem on Exam 1 is a good illustration (it involves the Fundamental Theorem of Calculus, which was current material, as well as l'Hospital's rule, which was from prior sections).

## 2. Exam II

2.1. **Section 5.7 and Appendix G.** These sections cover the partial fractions technique for integrating rational functions.

You should be able to:

- Put an arbitrary rational function into the proper form to apply this technique
- Write the partial fraction expansion for **any** rational function in terms of the unknown parameters
- For **any** rational function, set up the system of linear equations that the unknown coefficients must satisfy.
- For a computationally simple case, solve the system of linear equations (i.e., a system of two equations in two unknowns or three equations in three unknowns) to find the unknown coefficients in the partial fraction expansion.
- Integrate **any** given partial fraction expansion that includes all coefficients.

Example: Find

$$\int \frac{2x^3 + 3x^2 - 3x - 1}{2x^2 + 3x - 2}$$

Solution: The degree of the numerator is greater, so use long division to write the integrand as the sum of a quotient and remainder:

$$\frac{2x^3 + 3x^2 - 3x - 1}{2x^2 + 3x - 2} = (2x+1) + \frac{1}{x^2 + x - 2} = (2x+1) + \frac{1}{(x-1)(x+2)}$$

The partial fractions expansion of the remainder is (Case I in the text):

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

Multiplying both sides by $(x-1)(x+2)$ gives

$$1 = A(x+2) + B(x-1) = (Ax + Bx) + (2A - B) = x(A+B) + (2A-B)$$

Now equate the coefficients of like powers of $x$ on both sides to obtain two equations in two unknowns:

$$\begin{array}{rcrcl} A & + & B & = & 0 \\ 2A & - & B & = & 1 \end{array}$$

Subtracting the first equation from the second gives:

$$3A = 1$$

so $A = 1/3$. Then by substitution,

$$0 = A + B = \frac{1}{3} + B$$

so $B = -1/3$, and we can write

$$\frac{2x^3 + 3x^2 - 3x - 1}{2x^2 + 3x - 2} \;=\; (2x+1) + \frac{1}{3(x-1)} - \frac{1}{3(x+2)}$$

Integrating the left hand side, we get

$$\int \frac{2x^3 + 3x^2 - 3x - 1}{2x^2 + 3x - 2}\,dx \;=\; \int (2x+1)\,dx + \frac{1}{3}\int \frac{dx}{(x-1)} - \frac{1}{3}\int \frac{dx}{(x+2)}$$

$$= \frac{2x^2}{2} + x + \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+2) + C$$

2.2. **Section 5.8.** Since we do not have access to a CAS during the exam, no questions on this material are possible. However, it is possible to supply an entry from the table of integrals and a function and ask how to use the table entry. Here is an example: Given that for $a > 0$,

$$\int \sqrt{a^2 - u^2}\,dx \;=\; \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

find

$$\int \sqrt{1 - 4x^2}\,dx$$

Solution: Let $u = 2x$. Then $dx = du/2$ and using the substitution rule, the integral is

$$\frac{1}{2}\int \sqrt{1 - u^2}\,du$$

Identify $a^2$ with 1, so $a = 1$ ($-1$ is ruled out by the condition $a > 0$), and by substitution the result is

$$\int \sqrt{1 - 4x^2}\,dx \;=\; \frac{2x}{2}\sqrt{1 - 4x^2} + \frac{1}{2}\sin^{-1}\frac{2x}{1} + C$$

$$= x\sqrt{1 - 4x^2} + \frac{1}{2}\sin^{-1} 2x + C$$

2.3. **Section 5.9.** As with quiz4, I will supply the following formulas:

Midpoint Rule

$$\int_a^b f(x)\,dx \approx M_n = \Delta x\,[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)]$$

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad \overline{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$$

$$|E_M| \le \frac{K((b-a)^3}{24n^2}, \quad \text{where} \quad |f''(x)| \le K, \quad a \le x \le b$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\,[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x$$

$$|E_T| \le \frac{K((b-a)^3}{12n^2}, \quad \text{where} \quad |f''(x)| \le K, \quad a \le x \le b$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx S_n = \frac{\Delta x}{3}\,[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots$$
$$+2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$\Delta x = \frac{b-a}{n}$$

$$|E_S| \le \frac{K((b-a)^5}{180n^4}, \quad \text{where} \quad |f^{(4)}(x)| \le K, \quad a \le x \le b$$

You should be able to do problems similiar to those on quiz 4. All problems will be chosen so that the computations can be done by hand.

1) (8 pts) Use the Trapezoidal rule with $n = 4$ to approximate

$$\int_0^4 x\,dx$$

What is the bound for the maximum absolute error in this case?

2) (8 pts) If the midpoint rule with $n = 6$ is used to estimate

$$\int_0^4 x^2\,dx$$

what is the upper bound for the absolute error?

3) (9 pts) Use Simpson's rule with $n = 4$ to approximate

$$\int_0^{\pi/2} \cos 4x \, dx$$

What is the bound for the maximum absolute error in this case?

2.4. **Section 5.10.**

- You should be able to recognize and set up both Type I (p.424) and Type II (p.427) improper integrals.
- You should be able to use [2] on page 427 ($\int_1^\infty dx/x^p$)
- You should know how to use the comparison test (p.429) for convergence and divergence.

The problems on the takehome assignment are typical.

2.5. **Section 6.1.** You should know how to apply the definitions ([1] on p.441) and ([2] on p.442) to find the area between two curves. Often this will involve finding the points of intersection as in the classroom examples from 3/15.

You should be able to apply the formula for parametric curves on p.445,

$$A = \int_a^b y \, dx \; = \; \int_\alpha^\beta g(t) f'(t) dt$$

as in Example 6 and the example from class on 3/15.