MA126 Quiz 8

Name:

1) (8 pts) Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

$$a_n = \frac{3+5n^2}{n+n^2}$$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3+5n^2}{n+n^2} = \lim_{n\to\infty} \frac{(3+5n^2)(1/n^2)}{(n+n^2)(1/n^2)}$

$= \lim_{n\to\infty} \frac{\frac{3}{n^2}+5}{\frac{1}{n}+1} = \frac{5}{1} = 5$

$\therefore$ converges

2) (8 pts) Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

$$a_n = \frac{\ln n}{n}$$

Use the fact that if $\lim_{x\to\infty} f(x)$ converges, then $\lim_{n\to\infty} f(n)$, $n = 1, 2, \ldots$ converges.

$\lim_{x\to\infty} \frac{\ln x}{x} \Rightarrow \frac{\infty}{\infty}$ By L'Hospital's rule, $\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$

(OVER)

3) (9 pts) Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$3 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \cdots$$

Write the sequence as

$$3 - \frac{10}{3}\left[1 - \frac{2}{3} + \left(\frac{2}{3}\right)^2 - \cdots\right]$$

$$= 3 - \frac{10}{3}\left[\frac{1}{1-(-2/3)}\right] = 3 - \frac{10}{3}\cdot\frac{3}{5} = 1$$

↑ geometric series $r = -\frac{2}{3}$