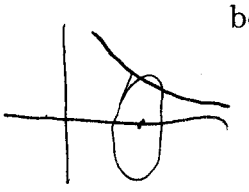
MA126 Quiz 6

Name:

1) (8 pts) Find the volume of the solid created by revolving the graph of

$$f(x) = \frac{1}{x}$$

between $x = 1$ and $x = 2$ about the x-axis.



Disk method:

$$V = \int_a^b \pi (f(x))^2 \, dx = \pi \int_1^2 \frac{dx}{x^2} = \pi \left(-\frac{1}{x} \right) \Big|_1^2 = \pi \left(-\frac{1}{2} - (-1) \right)$$

$$= \frac{\pi}{2}$$

2) (8 pts) Find the exact length of the curve

$$x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \le t \le 1$$

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt = \int_0^1 \sqrt{(6t)^2 + (6t)^2} \, dt$$

$$= \int_0^1 \sqrt{2 \cdot 36 \cdot t^2} \, dt = \int (6\sqrt{2}) \sqrt{t^2} \, dt = 6\sqrt{2} \int_0^1 t \, dt = 6\sqrt{2} \, \frac{t^2}{2} \Big|_0^1$$

$$= 6\sqrt{2}$$

(OVER)

3) (9 pts) Find the average value of

$$f(x) = (\sin x)^2$$

over the interval from 0 to 2π.

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2\pi}\int_0^{2\pi} \sin^2 x\,dx$$

$$= \frac{1}{2\pi}\,\frac{1}{2}\int_0^{2\pi}(1-\cos 2x)\,dx = \frac{1}{2\pi}\left[\frac{1}{2}x\Big|_0^{2\pi} - \frac{1}{2}\cdot\frac{1}{2}\sin 2x\Big|_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\pi - 0\right] = \frac{1}{2}$$