MA126 Quiz 5

Name:

1) (8 pts) Evaluate the following integral (if it converges)

$$\int_{\frac{1}{2}}^{1} \frac{dx}{\sqrt{2x-1}}$$

$$= \lim_{t \to \frac{1}{2}^+} \int_t^1 (2x-1)^{-1/2}\,dx \quad = \lim_{t \to 1/2} \frac{(2x-1)^{1/2}}{(1/2)} \cdot \frac{1}{2} = 1$$

2) (8 pts) Use the Comparison Theorem to determine whether the following integral is convergent or divergent when a is a positive constant. (**DO NOT** evaluate the integral):

$$\int_1^{\infty} a \cdot \frac{\cos^2 x}{1+x^2}\,dx \qquad \cos^2 x \le 1$$

$$\int_1^{\infty} a \frac{\cos^2 x}{1+x^2}\,dx = a\int_1^{\infty} \frac{\cos^2 x}{1+x^2}\,dx \le \int_1^{\infty} \frac{1}{1+x^2}\,dx \le \int_1^{\infty} \frac{dx}{x^2}$$

↑ Convergent P-Series : $p > 1$

(OVER)

3) (9 pts) Determine whether the following improper integral converges when a is a positive constant. If the integral does converge, find its value.

$$\int_0^\infty \frac{dx}{(\sqrt{1+ax})^3}$$

$$\int_0^\infty (1+ax)^{-3/2}\,dx$$

$$= \lim_{t\to\infty} \int_0^t (1+ax)^{-3/2}\,dx$$

$$= \lim_{t\to\infty} \frac{(1+ax)^{-1/2}}{(-1/2)} \cdot \frac{1}{a} \Bigg|_0^t$$

$$= \lim_{t\to\infty} \left(\frac{2}{a} - \frac{1}{\sqrt{1+at}} \cdot \frac{1}{2a} \right) = \frac{2}{a}$$

Converges