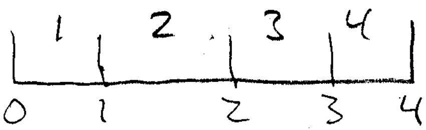
MA126 Quiz 4

Name: KEY

1) (8 pts) Use the Trapezoidal rule with $n = 4$ to approximate

$$\int_0^4 x\,dx$$

What is the bound for the maximum absolute error in this case?



$\Delta x = 1$

$T_4 = \frac{1}{2}\left[f(0)+2f(1)+2f(2)+2f(3)+f(4)\right]$

$= \frac{1}{2}\left[0+2\cdot 1+2\cdot 2+2\cdot 3+4\right]$

$= 8$

$f''(x) = 0$ so the error is zero

2) (8 pts) If the midpoint rule with $n = 6$ is used to estimate

$$\int_0^4 x^2\,dx$$

what is the upper bound for the absolute error?

$|E_M| \leq \frac{K(b-a)^3}{24n^2}$

$n=6 \quad b-a=4$

$|f''(x)| = |2| \leq K \Rightarrow$ Let $K=2$

$< \frac{2(4)^3}{24(36)} = \frac{8}{54} = .148$

(OVER)

3) (9 pts) Use Simpson's rule with $n = 4$ to approximate

$$\int_0^{\pi/2} \cos 4x \, dx$$

What is the bound for the maximum absolute error in this case?

$0 \quad \frac{\pi}{8} \quad \frac{\pi}{4} \quad \frac{3\pi}{8} \quad \frac{\pi}{2}$

Coefficient: $1 \quad 4 \quad 2 \quad 4 \quad 1$

$\Delta x = \frac{\pi}{8} = \frac{\frac{\pi}{2} - 0}{4}$

$S_4 = \frac{1}{3}\left(\frac{\pi}{8}\right)\left[\cos 0 + 4\cos\frac{\pi}{2} + 2\cos\pi + 4\cos\frac{3\pi}{2} + \cos 2\pi\right]$

$\frac{\pi}{24}\left[1 + 4(0) + 2(-1) + 4(0) + 1\right]$

$= \frac{\pi}{24}[0] = 0$

$f'(x) = -4\sin 4x$

$f''(x) = -16\cos x$

$f'''(x) = 64\sin x$

$f''''(x) = 256\cos x$

$|E_s| \le \frac{256\left(\frac{\pi}{2}\right)^5}{180 \cdot 256}$

Midpoint Rule

$$\int_a^b f(x)\,dx \approx M_n = \Delta x\,[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)]$$

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad \overline{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$$

$$|E_M| \le \frac{K((b-a)^3}{24n^2}, \quad \text{where} \quad |f''(x)| \le K, \quad a \le x \le b$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\,[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x$$

$$|E_T| \le \frac{K((b-a)^3}{12n^2}, \quad \text{where} \quad |f''(x)| \le K, \quad a \le x \le b$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx S_n = \frac{\Delta x}{3}\,[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots$$

$$+2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$\Delta x = \frac{b-a}{n}$$

$$|E_S| \le \frac{K((b-a)^5}{180n^4}, \quad \text{where} \quad |f^{(4)}(x)| \le K, \quad a \le x \le b$$