

MA126 Quiz 2

Name: KEY

1) (8 pts) Evaluate the indefinite integral

$$\int \sqrt{\cot x} \csc^2 x \, dx$$

Let $u = \cot x$ Then $\frac{du}{dx} = -\csc^2 x \Rightarrow dx = -\frac{du}{\csc^2 x}$

Now $\int \sqrt{\cot x} \csc^2 x \, dx = -\int \sqrt{u} \, du = -\frac{2}{3} u^{3/2} = -\frac{2}{3} (\cot x)^{3/2}$

2) (8 pts) Evaluate the definite integral

$$\int_{-1}^1 \frac{\sin x}{\sqrt{1+x^6}} \, dx = 0$$

$$\frac{\sin(-x)}{\sqrt{1+(-x)^6}} = -\frac{\sin(x)}{\sqrt{1+x^6}} \quad \text{so the integrand is an odd}$$

function. The integral of any odd function from $-a$ to a for $a > 0$ is zero

(OVER)

3) (9 pts) Evaluate the integral

$$\int_{-2}^2 (x+3)\sqrt{4-x^2} dx = 6\pi$$

Write the integral as: $\int_{-2}^2 x\sqrt{4-x^2} dx + 3\int_{-2}^2 \sqrt{4-x^2} dx$



↑ semicircle, $r=2$

second term is: $3 \cdot 2\pi$

$$\text{Let } u = g(x) = 4 - x^2$$

$$\frac{du}{dx} = -2x, \quad dx = -\frac{du}{2x}$$

on substitution, the integral is:

$$-\frac{1}{2} \int_{4-(-2)^2}^{4-(2)^2} \sqrt{u} du = -\frac{1}{2} \int_0^0 \sqrt{u} du = 0$$

Alternatively, you could use the fact that

$$(-x)\sqrt{4-(-x)^2} = -(x\sqrt{4-x^2})$$

So the integrand is odd

and $\int_{-a}^a f(x) dx = 0$ for any continuous

odd function and any $a > 0$.