MA126 Quiz 1

Name: KEY

1) (8 pts) A particle moves back and forth along a straight line. Its velocity at time t is given by

$$v(t) = \sqrt{4-t^2}$$

What is the net change in the particle's position from $t = 0$ to $t = 2$?

Since $v(t) = s'(t)$ where s is the position, the net change theorem says the net change in position is

$$\int_0^2 s'(t)\,dt = \int_0^2 v(t)\,dt = \int_0^2 \sqrt{4-t^2}\,dt = \pi$$

← quadrant of a circle with $r=2$

2) (8 pts) Use the properties of definite integrals to show that

$$\int_{0.4}^{1} \frac{x^2}{1000}\,dx \geq \int_{0.4}^{1} \ln x\,dx$$

On $[0.4, 1]$, $\frac{x^2}{1000} > 0$ and $\ln x \leq 0$

so $\frac{x^2}{1000} > 0 \geq \ln x \Rightarrow \frac{x^2}{1000} \geq \ln x$ on the interval

By the comparison property,

$$\int_{0.4}^{1} \frac{x^2}{1000}\,dx \geq \int_{0.4}^{1} \ln x\,dx$$

(OVER)

3) (9 pts) Evaluate the integral

$$\int_0^1 \frac{4}{t^2+1}\,dt$$

$$= 4\int_0^1 \frac{dt}{t^2+1} = 4\tan^{-1} t\Big|_0^1$$

$$= 4(\tan^{-1}1 - \tan^{-1}0)$$

$$= 4\left(\frac{\pi}{4} - 0\right)$$

$$= \pi$$