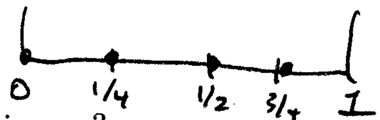
MA126 Exam 2 (version 1)

Name:

1) Use Simpson's rule with $n = 4$ to approximate

$$\int_0^1 \sqrt{x+1}\, dx$$



What is the bound for the maximum absolute error in this case?

$\Delta x = \frac{b-a}{4} = \frac{1}{4}$ $\quad \int_0^1 \sqrt{x+1}\, dx \approx \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left[f(0) + 4f\left(\frac{1}{4}\right) + 2f\left(\frac{1}{2}\right) + 4f\left(\frac{3}{4}\right) + f(1)\right]$

$= \frac{1}{12}\left[1 + 4\sqrt{\frac{5}{4}} + 2\sqrt{\frac{3}{2}} + 4\sqrt{\frac{7}{4}} + \sqrt{2}\right]$

$f'(x) = \frac{1}{2}(x+1)^{-1/2}$

$f''(x) = -\frac{1}{4}(x+1)^{-3/2}$

$f'''(x) = +\frac{3}{8}(x+1)^{-5/8}$

$f^{IV}(x) = -\frac{15}{16}(x+1)^{-7/8}$

maximized when $x=0$

Let $K = \frac{15}{16}$

$|E_s| \leq \frac{\frac{15}{16}(1-0)^5}{180(4^4)}$

2) Evaluate the following definite integral: (note that $\sin^2 x$ means $[\sin(x)]^2$)

$$\int_{-\pi}^{\pi} \sin^2(3\theta)\, d\theta$$

use $\sin^2(u) = \frac{1}{2}(1-\cos 2u)$

$\int_{-\pi}^{\pi} \sin^2 3\theta\, d\theta = \frac{1}{2}\int_{-\pi}^{\pi}(1-\cos(2\cdot 3\theta))\,d\theta =$

$\frac{1}{2}\int_{-\pi}^{\pi} d\theta - \frac{1}{2}\int_{-\pi}^{\pi}\cos 6\theta\, d\theta = \frac{\theta}{2}\Big|_{-\pi}^{\pi} - \frac{1}{2}\cdot\frac{1}{6}\sin 6\theta\Big|_{-\pi}^{\pi} = \frac{2\pi}{2} - 0 = \pi$

3) Evaluate the following improper integral if it exists (show all work):

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

write in two parts: $\int_{-\infty}^{0}\frac{dx}{1+x^2} + \int_{0}^{\infty}\frac{dx}{1+x^2}$

$= \lim_{t\to-\infty} \tan^{-1}x\Big|_t^0 + \lim_{t\to\infty}\tan^{-1}x\Big|_0^t$

$= \tan^{-1}0 - \lim_{t\to-\infty}\tan^{-1}t + \lim_{t\to\infty}\tan^{-1}t - \tan^{-1}0$

$= 0 - \left(-\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right) - 0 = \pi$

4) Find the area enclosed by the x-axis and the parametric curve defined by:

$$x = t^2, \quad y = \sin t, \quad 0 \le t \le \pi$$

$x = f(t) = t^2$
$y = g(t) = \sin t$

$$A = \int_0^{\pi} g(t) f'(t)\,dt = \int_0^{\pi} \sin t\,(2t)\,dt = 2\int_0^{\pi} t \sin t\,dt$$

let: $u = t$, Then $du = dt$

$dv = \sin t\,dt \quad V = -\cos t$

$$2\int_0^{\pi} t \sin t\,dt = 2\left[-t\cos t\Big|_0^{\pi} - \int_0^{\pi}(-\cos t)\,dt\right) = 2\left[\pi + \sin t\Big|_0^{\pi}\right] = 2\pi$$

5) Find the following integral, if it exists:

$$\int_1^2 \frac{dx}{(1-x)^3}$$

$$= \lim_{t\to 1^+} \int_t^2 \frac{dx}{(1-x)^3} = \lim_{t\to 1^+} -\frac{1}{2(1-x)^2}\Bigg]_t^2 = -\frac{1}{2} + \underbrace{\lim_{t\to 1^+} \frac{1}{2(1-x)^2}}_{\to\infty}$$

Integral does not EXIST

6) Use the Trapezoidal rule with $n = 4$ to approximate

$$\int_0^{\pi} \sin 2x\,dx \qquad \Delta x = \frac{b-a}{n} = \frac{\pi}{4}$$

What is the bound for the maximum absolute error in this case?

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$2x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin 2x$	0	1	0	-1	0

$$\int_0^{\pi} \sin 2x\,dx \approx \left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)\left[\sin(0) + 2\sin\left(\frac{\pi}{2}\right) + 2\sin\pi + 2\sin\left(\frac{3\pi}{2}\right) + \sin(2\pi)\right]$$

$$= \frac{\pi}{8}\left[0 + 2\cdot 1 + 2\cdot 0 + 2\cdot -1 + 0\right] = 0$$

$|f'(x)| = 2\cos 2x$
$|f''(x)| = |4\sin 2x| \quad$ max of $|\sin 2x|$ is 1
$K = 4$

$$|E| \le \frac{4(\pi-0)^2}{12\cdot 4^2} = \frac{\pi^2}{48}$$

7) Write out the form of the partial fractions expansion of the following function. **DO NOT** determine the numerical values of the coefficients.

$$f(x) = \frac{x^5 - x^4 + 2x^3 - 2x^2 + 2x}{(x^2+1)^2} \leftarrow$$

$b^2 - 4ac = 0^2 - 4 < 0$
$\therefore$ irreducible
Repeated $\Rightarrow$ case $\underline{\text{IV}}$

$$\frac{x^5 - x^4 + 2x^3 - 2x^2 + 2x}{(x^2+1)^2} = \frac{A_1 x + B_1}{(x^2+1)} + \frac{A_2 x + B_2}{(x^2+1)^2}$$

8) Evaluate the following integral if it exists. Assume that t is a positive constant. Show all work.

$$\int_1^\infty \frac{2 + e^{-tx}}{x}\, dx$$

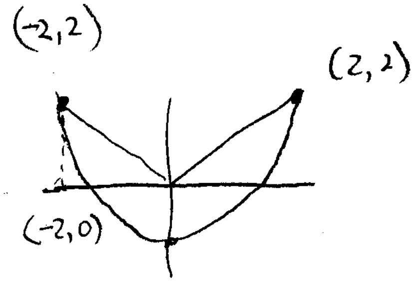
write as:

$$\int_1^\infty \left(\frac{2}{x} + \frac{e^{-tx}}{x}\right) dx = \int_1^\infty \frac{2}{x} dx + \int_1^\infty \frac{e^{-tx}}{x} dx$$

$\uparrow$

$$\lim_{t \to \infty} 2 \ln|x| \Big|_1^t \to \infty$$

So the integral does not EXIST



9) Find the area of the region bounded by the curves

$$y = |x| \quad \text{and} \quad y = x^2 - 2$$

$$A = \int_{-2}^{0} (-x - x^2 + 2)\,dx + \int_{0}^{2} (x - x^2 + 2)\,dx$$

$$= \int_{-2}^{0} -x\,dx + \int_{0}^{2} x\,dx + \int_{-2}^{2} (2 - x^2)\,dx = -\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{2} - \frac{x^3}{3}\Big|_{-2}^{2} + 2x\Big|_{-2}^{2}$$

$$0 - \left(-\frac{4}{2}\right) + \frac{4}{2} - \frac{16}{3} + 8 = \frac{20}{3}$$

10) Evaluate the following integral using partial fractions (including the numerical value of the coefficients and the result of the integration). Show all work.

$$\int \frac{x+1}{x^2 - 2x + 1}\,dx = \int \frac{x+1}{(x-1)^2}$$

Case II:

$$\frac{x+1}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2}$$

multiply both sides by $(x-1)^2$:

$$x + 1 = A(x-1) + B$$

Equate coefficients of like powers of x,

$$x = Ax \Rightarrow A = 1$$

$$1 = -A + B \Rightarrow B = 1 + A = 2$$

$$\int \frac{x+1}{(x-1)^2}\,dx = \int \left(\frac{1}{(x-1)} + \frac{2}{(x-1)^2}\right) dx = \int \frac{dx}{x-1} + 2\int \frac{dx}{(x-1)^2}$$

$$= \ln|x-1| + 2\,\frac{(x-1)^{-1}}{-1}$$

$$= \ln|x-1| - \frac{2}{x-1} + C$$