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The interpretation is that $F' = f$, that is, we write

$$\int f(x)dx = F(x) \quad \text{if and only if} \quad F'(x) = f(x)$$

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We should be able to do this for any of the entries in the table.

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Then

$$\frac{d}{dx}\left(\frac{1}{\ln a} e^{\ln a \cdot x} + C\right) = \frac{1}{\ln a} \ln a \cdot e^{\ln a \cdot x} = a^x$$

Question 1

Find the most general indefinite integral:

$$\int (\sin x + \sinh x)\, dx$$

1. $\cos x + \cosh x + C$
2. $-\cos x + \cosh x + C$
3. $\cos x - \cosh x + C$
4. $-\cos x - \cosh x + C$
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6. None of the choices given

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2. $-\cos x + \cosh x + C$

Question 2

Find the most general indefinite integral:

$$\int (1 + \tan^2 x)\, dx$$

1. $\sec^2 x + C$
2. $\sec x \tan x + C$
3. $\tan x + C$
4. $\tan^2 x + C$
5. $x + \tan^2 x + C$
6. None of the choices given

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6. None of the choices given

3. $\tan x + C$ (rewrite the integrand as $\sec^2 x dx$)

Question 3

Find the most general indefinite integral:

$$\int x^n dx$$

1. $\frac{x^{n+1}}{n} + C$
2. $\frac{x^{n-1}}{n-1} + C$
3. $\frac{x^n}{n+1} + C$
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4 $\frac{x^{n+1}}{n+1} + C$

The Net Change Theorem

If f represents the rate of change of some quantity,

$$\int_a^b f(x)dx = F(b) - F(a)$$

represents the **net change** of that quantity.

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The net change from $t = 0$ to $t = 2$ is

$$\int_0^2 e^{-t} dt \quad = \quad -e^{-2} - \left(-e^{-0}\right) = 1 - e^{-2} = 0.864$$

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The net change in position from $t = 0$ to $t = 4$ is

$$\int_0^2 (3 + 4t)dt \quad = \quad (3 \cdot 4 + 2 \cdot 4^2) - (3 \cdot 0 + 2 \cdot 0^2) = 44\text{ft}$$

Question 4

An object is released at a height of $3000ft$ and falls with a constant acceleration of $32ft/sec^2$.

What is the net change in the velocity v of the object during the first 3 seconds?

1. $\int_0^3 32t dt$
2. $\int_1^3 32 dt$
3. $\int_0^3 32 dt$
4. $\int_0^3 32t^2 dt$
5. $\int_0^3 16t^2 dt$
6. None of the choices given

Question 4

An object is released at a height of $3000ft$ and falls with a constant acceleration of $32ft/sec^2$.

What is the net change in the velocity v of the object during the first 3 seconds?

1. $\int_0^3 32tdt$
2. $\int_1^3 32dt$
3. $\int_0^3 32dt$
4. $\int_0^3 32t^2dt$
5. $\int_0^3 16t^2dt$
6. None of the choices given

3. The net change is $\int_0^3 32dt = 32(3) - 32(0) = 96$