The Definite Integral

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We now introduce a related concept in terms of the area under the graph of a function f know as the *definite integral* of f

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If f is defined on an interval $[a, b]$, the **definite integral of** f **from** a **to** b is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x$$

if the limit exists. Some explanation of the notation is required.

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We define x_i^* to be any point in the i^{th} interval:
$x_{i-1} \leq x_i^* \leq x_i$

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The precise statement of the limit process is as follows:

For every $\epsilon > 0$, it is possible to find an integer N such that

$$\left| \int_a^b f(x)dx - \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x \right| < \epsilon$$

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Note that unlike an antiderivative, the definite integral, if it exists, is a **number**

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