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Example:

$F(x) = x^2 - 3x - 5$ is an antiderivative of $f(x) = 2x - 3$ on $\mathbb{R}$

In fact, for any $C \in \mathbb{R}$,

$F(x) = x^2 - 3x + C$ is an antiderivative of $f(x) = 2x - 3$ on $\mathbb{R}$

Antiderivatives

Recall Corollary 7 of Section 4.2:

If $f'(x) = g'(x)$ for all x on in interval (a,b), then

$$f(x) = g(x) + C \quad \text{on} \quad (a,b)$$

Antiderivatives

Recall Corollary 7 of Section 4.2:

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Applied to antiderivatives, it says that any two antiderivatives of a function differ by a constant:

If $F'(x) = G'(x)$ for all x on in interval (a, b), then

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Antiderivatives

It is customary to write the most general antiderivative in the form of a specific antiderivative plus a constant C

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In the case of the previous example, the most general antiderivative of

$$f(x) = 2x - 3$$

is

$$F(x) = x^2 - 3x + C$$

Question 1

Which of the following is an antiderivative of

$$f(x) = x^4$$

1. $f(x) = x^5$
2. $f(x) = 4x^5$
3. $f(x) = x^5/5$
4. $f(x) = 5x^4$
5. $f(x) = x^4/4$
6. None of the above

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3.

Question 2

Which of the following is an antiderivative of

$$f(x) = 3x^2 + 2x + 1$$

1. $f(x) = x^3 + x^2 + x + 2$
2. $f(x) = x^3 + x^2 + 2x + 1$
3. $f(x) = 2x^3 + x^2 + x + 2$
4. $f(x) = x^3 + x^2 + x - 1$
5. 1 and 4
6. None of the above

Question 2

Which of the following is an antiderivative of

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5. (1 and 4)

Differential Equations

An equation that involves derivatives of a function is called a **differential equation**:

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The solution to a differential equation, if it has one, can be written as a function plus a constant C

$$f(x) = 3x - 2x^2 + C$$

Differential Equations

Alternatively, a differential equation may include an extra condition that uniquely determines C:

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impies that

$$3 \cdot 1 - 2 \cdot 1^2 + C = 0 \quad \text{so} \quad 1 + C = 0 \quad \text{and} \quad C = -1$$

and

$$f(x) = 3x - 2x^2 - 1$$

Question 3

Find a function satisfying

$$f'(x) = 2x - 8 \quad \text{and} \quad f(0) = 1$$

1. $f(x) = x^2 - 8x$
2. $f(x) = x^2 - 8x + 2$
3. $f(x) = x^2 - 8x + 4$
4. $f(x) = x^2 - 8x + 3$
5. $f(x) = x^2 - 8x + 1$
6. None of the above

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Straight Line Motion

The velocity of a particle is the time derivative of position. Given a velocity function,

$$v(t) = 12 + t$$

the position function is an antiderivative of $v(t)$:

$$s(t) = 12t + \frac{t^2}{2} + C$$

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If we are given an initial position, the constant C is determined: Suppose $s(0) = 12$. Then

$$s(t) = 12t + \frac{t^2}{2} + 12$$

Question 4

A particle moves in a straight line with velocity function

$$v(t) = 3t^2 + 6t + 1$$

Find the position function $s(t)$ given that $s(0) = 3$

1. $s(t) = t^3 + 3t^2 + t + 5$
2. $s(t) = t^3 + 3t^2 + t + 3$
3. $s(t) = t^3 + 3t^2 + t + 1$
4. $s(t) = t^3 + 3t^2 + t + 2$
5. $s(t) = t^3 + 3t^2 + t + 4$
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2 $s(t) = t^3 + 3t^2 + t + 3$

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Sometimes ingenuity is required to find antiderivatives:

Find an antiderivative of

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Solution:

$$x^2 \sin x$$

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In some cases it may not be possible to find an antiderivative:

Find an antiderivative of

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Solution: This function has no antiderivative.