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Both x and  $\sin x$  are continuous at x = 0, so the limits are the function values. Both are zero so

$$\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$$

L'Hospital's rule says that in this situation, we can replace the numerator and denominator by their derivatives:

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}$$

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In section 3.3, this limit was shown to be 1 by a geometric argument (p.191)

L'Hospital's rule gives us an eaiser way to determine this and other limits.

L'Hospital's rule also works for limits at  $\infty$  or  $-\infty$ :

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It also works for right hand limits:

$$\lim_{x \to a^+} \frac{f(x)}{g(x)} = \lim_{x \to a^+} \frac{f'(x)}{g'(x)}$$

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$$\lim_{x \to a^+} \frac{f(x)}{g(x)} = \lim_{x \to a^+} \frac{f'(x)}{g'(x)}$$

and for left hand limits:

$$\lim_{x \to a^-} \frac{f(x)}{g(x)} = \lim_{x \to a^-} \frac{f'(x)}{g'(x)}$$

Evaluate the following limit:

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

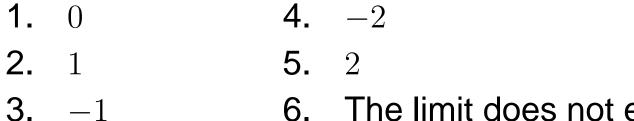
- **1.** 0 **4.** -2
- **2.** 1 **5.** 2

**3.** -1

6. The limit does not exist

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#### 1. The limit is 0

L'Hospital's Rule also works for the following indeterminite form(s):



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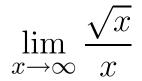
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L'Hopital's rule in this case says

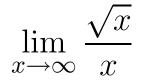
$$\lim_{x \to \infty} \frac{x}{x^2} = \lim_{x \to \infty} \frac{1}{2x} = 0$$





- **1.** 0 **4.** -2
- 2. 1
  3. −1
  5. 2
  6. T
  - The limit does not exist

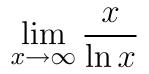




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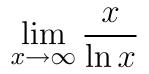
#### **1.** The limit is 0

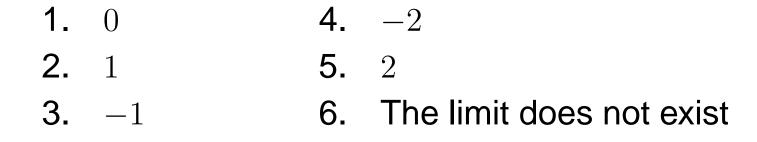




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6. The limit does not exist  $\left(\frac{x}{\ln x} \to \infty \text{ as } x \to \infty\right)$ 

L'Hospital's Rule may have to be applied more than once. Suppose we have the limit:

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A second application gives:

$$\lim_{x \to \infty} \frac{2x}{e^x} = \lim_{x \to \infty} \frac{2}{e^x} = 0$$

Evaluate the following limit:

$$\lim_{x \to 0} \frac{x^2}{1 - \cos x}$$

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- **2.** 1 **5.** 2

**3.** -1

6. The limit does not exist

Evaluate the following limit:

$$\lim_{x \to 0} \frac{x^2}{1 - \cos x}$$

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- **2.** 1 **5.** 2
- 3. -1 6. The limit does not exist

5. The limit is 2



$$\lim_{x \to \infty} \frac{x^2}{e^x}$$

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- 3. -1 6. The limit does not exist



$$\lim_{x \to \infty} \frac{x^2}{e^x}$$

- **1.** 0 **4.** -2
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- 3. -1 6. The limit does not exist

**1.** The limit is 0

Evaluate the following limit:

$$\lim_{x \to 0^+} \frac{\ln x}{1/x}$$

- **1.** 0 **4.** -2
- **2.** 1 **5.** 2

**3.** -1

6. The limit does not exist

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$$\lim_{x \to 0^+} \frac{\ln x}{1/x}$$

**1.** 0 **4.** -2

3. -1 6. The limit does not exist

**1.** The limit is 0

2.

#### **Indeterminate Products**

Sometimes a limit involves a product of the form  $0 \cdot \infty$ .

 $\lim_{x \to 0^+} \ln x \cdot \sin x = -\infty \cdot 0$ 

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We cannot apply L'Hopital's rule directly, but we can transform to an  $\infty/\infty$  form:

$$\ln x \cdot \sin x = (\ln x \cdot \sin x) \left(\frac{1/\sin x}{1/\sin x}\right) = \frac{\ln x}{1/\sin x} = \frac{-\infty}{\infty}$$

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Now we apply L'Hopital's rule twice to obtain:

$$\lim_{x \to 0^+} \frac{\ln x}{1/\sin x} = \lim_{x \to 0^+} \frac{1/x}{-\cos x/\sin^2 x} = 0$$

Evaluate the following limit:

$$\lim_{x \to -\infty} x \cdot e^x$$

- **1.** 0 **4.** -2
- **2.** 1 **3.** -1
- **5.** 2
  - 6. The limit does not exist

Evaluate the following limit:

$$\lim_{x \to -\infty} x \cdot e^x$$

1. 0
 4. −2
 2. 1
 5. 2
 3. −1
 6. The limit does not exist

**1.** The limit is 0