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Both x and $\sin x$ are continuous at $x = 0$, so the limits are the function values. Both are zero so

$$\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$$

L'Hospital's Rule

L'Hospital's rule says that in this situation, we can replace the numerator and denominator by their derivatives:

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}$$

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In section 3.3, this limit was shown to be 1 by a geometric argument (p.191)

L'Hospital's rule gives us an eaiser way to determine this and other limits.

L'Hospital's Rule

L'Hospital's rule also works for limits at ∞ or $-\infty$:

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It also works for right hand limits:

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It also works for right hand limits:

$$\lim_{x\to a^+} \frac{f(x)}{g(x)} = \lim_{x\to a^+} \frac{f'(x)}{g'(x)}$$

and for left hand limits:

$$\lim_{x\to a^-} \frac{f(x)}{g(x)} = \lim_{x\to a^-} \frac{f'(x)}{g'(x)}$$

Question 1

Evaluate the following limit:

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

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This form often arises from limits at infinity:

$$\lim_{x\to\infty}\frac{x}{x^2} = \frac{\infty}{\infty}$$

L'Hopital's rule in this case says

$$\lim_{x\to\infty}\frac{x}{x^2} = \lim_{x\to\infty}\frac{1}{2x} = 0$$

Question 2

Evaluate the following limit:

$$\lim_{x \to \infty} \frac{\sqrt{x}}{x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

Question 2

Evaluate the following limit:

$$\lim_{x \to \infty} \frac{\sqrt{x}}{x}$$

1.	0	4.	-2
2.	1	5.	2
3.	-1	6.	The limit does not exist

1. The limit is 0

Question 3

Evaluate the following limit:

$$\lim_{x \to \infty} \frac{x}{\ln x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

Question 3

Evaluate the following limit:

$$\lim_{x \to \infty} \frac{x}{\ln x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

6. The limit does not exist $\left(\frac{x}{\ln x} \to \infty \text{ as } x \to \infty\right)$

Indeterminate Forms

L'Hospital's Rule may have to be applied more than once. Suppose we have the limit:

$$\lim_{x \to \infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$$

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A second application gives:

$$\lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$$

Question 4

Evaluate the following limit:

$$\lim_{x \to 0} \frac{x^2}{1 - \cos x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

Question 4

Evaluate the following limit:

$$\lim_{x \to 0} \frac{x^2}{1 - \cos x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

5. The limit is 2

Question 5

Evaluate the following limit:

$$\lim_{x \to \infty} \frac{x^2}{e^x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

Question 5

Evaluate the following limit:

$$\lim_{x \to \infty} \frac{x^2}{e^x}$$

1.	0	4.	-2
2.	1	5.	2
3.	-1	6.	The limit does not exist

1. The limit is 0

Question 6

Evaluate the following limit:

$$\lim_{x \to 0^+} \frac{\ln x}{1/x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

Question 6

Evaluate the following limit:

$$\lim_{x \to 0^+} \frac{\ln x}{1/x}$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

1. The limit is 0

Indeterminate Products

Sometimes a limit involves a product of the form $0 \cdot \infty$.

$$\lim_{x \to 0^+} \ln x \cdot \sin x \;=\; -\infty \cdot 0$$

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We cannot apply L'Hopital's rule directly, but we can transform to an ∞/∞ form:

$$\ln x \cdot \sin x \ = \ (\ln x \cdot \sin x) \left(\frac{1/\sin x}{1/\sin x} \right) \ = \ \frac{\ln x}{1/\sin x} = \frac{-\infty}{\infty}$$

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$$\ln x \cdot \sin x \;=\; (\ln x \cdot \sin x)\left(\frac{1/\sin x}{1/\sin x}\right) \;=\; \frac{\ln x}{1/\sin x} = \frac{-\infty}{\infty}$$

Now we apply L'Hopital's rule twice to obtain:

$$\lim_{x \to 0^+} \frac{\ln x}{1/\sin x} \;=\; \lim_{x \to 0^+} \frac{1/x}{-\cos x/\sin^2 x} = 0$$

Question 7

Evaluate the following limit:

$$\lim_{x \to -\infty} x \cdot e^x$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

Question 7

Evaluate the following limit:

$$\lim_{x \to -\infty} x \cdot e^x$$

1. 0
2. 1
3. -1
4. -2
5. 2
6. The limit does not exist

1. The limit is 0