The First Derivative Test

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So it can apply to either kind of critical number:

- $f'(c) = 0$
- $f'(c)$ does not exist

Example 1

If

$$f(x) = |x|$$

then f has a critical value at $x = 0$ ($f'(0)$ does not exist)

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The first derivative changes from negative to positive.

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The first derivative changes from negative to positive.

The first derivative test says $f(x) = |x|$ has a local minimum at 0

Example 2

If

$$f(x) = -x^2$$

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The derivative changes from positive to negative

The first derivative test says $f(x) = -x^2$ has a local maximum at 0

Example 3

If

$$f(x) = x^3$$

then f has a critical value at $x = 0$ ($f'(0) = 0$)

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The first derivative does not change sign at 0

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To the right of zero, $f'(x) = 3x^2$ is also positive

The first derivative does not change sign at 0

The first derivative test says $f(x) = x^3$ has neither a local minimum nor a local maximum at 0

Question 1

Apply the first derivative to the critical values of
$f(x) = x^2 - 4x + 2$

1. f has a local minimum at $x = 2$
2. f has a local maximum a $x = 2$
3. f has neither a local maximum nor a local minimum at $x =$
4. The test does not apply

Question 1

Apply the first derivative to the critical values of $f(x) = x^2 - 4x + 2$

1. f has a local minimum at $x = 2$
2. f has a local maximum a $x = 2$
3. f has neither a local maximum nor a local minimum at $x =$
4. The test does not apply

2. $f'(2) = 0$. f has a local minimum at $x = 2$

Increasing/Decreasing Functions

If the derivative of a function is positive or negative on an interval I,

- f is increasing on I if f' is positive
- f is decreasing on I if f' is negative

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f' must have the same sign everywhere on I

Concave up/down

If the **second** derivative of a function is positive or negative on an interval I,

- f is concave up on I if f'' is positive
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f'' must have the same sign everywhere on I

Concave up (down) means all tangent lines are below (above) the graph of f

Question 2

Find the intervals where

$$f(x) = x^3 - 3$$

is concave up and down

1. up:$(-\infty, 0)$ down: $(0, \infty)$
2. up:$(-\infty, \infty)$
3. down:$(-\infty, \infty)$
4. up:$(0, \infty)$ down: $(-\infty, 0)$
5. up:$(0, \infty)$ down: nowhere
6. None of the above

Question 2

Find the intervals where

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Question 3

Find the intervals where

$$f(x) = x^2 - 2x + 3$$

is increasing and decreasing

1. inc:$(-\infty, 0)$ dec: $(0, \infty)$
2. inc:$(1, \infty)$ dec: $(-\infty, 1)$
3. inc:$(-\infty, \infty)$
4. inc:$(-1, \infty)$ dec: $(-\infty, -1)$
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2.