Mean Value Theorem

The mean value theorem states that, if a function f is continuous on a closed interval $[a, b]$, there has to be a point c between a and b (but not equal to either) where the tangent line is parallel to the secant line connecting $(a, f(a))$ and $(b, f(b))$.

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You absolutely must have a function that is continuous on an interval that includes the endpoints, or the theorem does not apply.

Rolle's Theorem

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The secant line from $(a, f(a))$ to $(b, f(b))$ has slope zero in this case, so the conclusion is that there is a c between a and b with $f'(c) = 0$.

Question 1

A function is continuous on $[1, 2]$ and has $f(1) = 3$ and $f(2) = 4$.

According to the mean value theorem,

1. $f'(c) = 1$ for some c in $(1, 2)$
2. $f'(c) = 0$ for some c in $(1, 2)$
3. $f'(c) = 1$ for some c in $[1, 2]$
4. $f'(c) = 1$ for some c in $(3, 4)$
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6. The theorem does not apply

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1. $f'(c) = 1 = (f(2) - f(1))/(2 - 1)$ for some c in $(1, 2)$

Question 2

A function is continuous on $(0, 2)$ and has $f(0) = f(2) = 1$.

According to Rolle's theorem,

1. $f'(c) = 0$ for some c in $(1, 2)$
2. $f'(c) = 0$ for some c in $(0, 2)$
3. $f'(c) = 0$ for some c in $[0, 2]$
4. $f'(c) = 0$ for some c in $(0, 1)$
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6. The theorem does not apply because $(0, 2)$ is not a closed interval

A Theorem

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This theorem establishes that the *converse* of that statement is also true:

If a function has derivative zero everywhere on an interval, then $f(x) = c$ on the interval.

A Corollary

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$$f(x) - g(x) - c \quad \text{on} \quad (a, b)$$

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It is important that f' and g' be defined everywhere on (a, b).

Example

Let $f(x) = x^2$ and $g(x) = x^2 + 3$ on $(-1, 1)$.

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The corollary says that f and g must differ by a constant, which is true.