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The interval (a, b) may have to be chosen to be very small.

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The function

$$f(x) = \cos x$$

has a **local minimum** at

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If we choose an interval containing π, say $(\pi/2, 3\pi/2)$,

$$f(c) = \cos(\pi) = -1 \leq \cos(x) \quad \text{for all } x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

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Again, the interval (a, b) may have to be chosen to be very small.

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$$f(x) = \sin x$$

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For every value of x in $(0, \pi)$,

$$f(c) = \sin(\pi/2) = 1 \geq \sin(x) \quad \text{for all } x \in (0, \pi)$$

Absolute Minimum, Maximum

Definition: Absolute Minimum

A function f has an **absolute minimum** (or **global minimum**) at $x = c$ if

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Absolute Minimum, Maximum

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The function

$$f(x) = -x^2$$

has an **absolute maximum** at $x = 0$ because

$$f(0) \geq f(x) \quad \text{for every } x \text{ in the domain of } f$$

Critical Number

Definition: Critical Number If f is a function and c is an element of the domain of f such that

$$f'(c) = 0$$

or

$$f'(c) \quad \text{does not exist}$$

then we say that c is a **critical number** of f.

Critical Number

Example: 0 is a critical number of

$$f(x) = | x |$$

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Critical Number

Theorem: If f has a local maximum or minimum at $x = c$, then x is a critical number of f.

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Theorem: (Fermat's Theorem) If f has a local maximum or minimum at $x = c$ and $f'(c)$ exists, then

$$f'(c) = 0$$

Extreme Value Theorem

Theorem: (Extreme Value Theorem) If f is continuous on a **closed** interval $[a, b]$ then f attains an absolute maximum $f(c)$ and absolute minimum value $f(d)$ at some values

$$c, d \in [a, b]$$

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Theorem: (Extreme Value Theorem) If f is continuous on a **closed** interval $[a, b]$ then f attains an absolute maximum $f(c)$ and absolute minimum value $f(d)$ at some values

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The interval $[a, b]$ has to be closed, that is, has to include its endpoints.

Closed Interval Method

To find the **absolute** maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

Find the value of f at all critical numbers of f in $[a, b]$

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- Find the value of f at all critical numbers of f in $[a, b]$
- Find the values of f at the endpoints of the interval: $f(a)$ and $f(b)$
- The largest value produced by the previous two steps is the absolute maximum; the smallest is the absolute minimum of f on $[a, b]$.

Closed Interval Method

Example: Find the absolute max and min of $f(x) = 2x^2$ on $[-1, 2]$.

$f'(x)$ is $4x$, which exists everywhere on $[-1, 2]$. So, there are no values where $f'(x)$ fails to exist and no critical numbers of that kind.

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$f'(x)$ is $4x$, which exists everywhere on $[-1, 2]$. So, there are no values where $f'(x)$ fails to exist and no critical numbers of that kind.

$f'(x) = 0$ at $x = 0$, so 0 is a critical number of f.

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Combining the values of f at critical numbers and endpoints, we have:

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The absolute max is 8, the absolute min is 0.