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The interval (a, b) may have to be chosen to be very small.

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has a **local minimum** at

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If we choose an interval containing π, say $(\pi/2, 3\pi/2)$,

$$f(c) = \cos(\pi) = -1 \leq \cos(x) \quad \text{for all } x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

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For every value of x in $(0, \pi)$,

$$f(c) = \sin(\pi/2) = 1 \geq \sin(x) \quad \text{for all } x \in (0, \pi)$$

Absolute Minimum, Maximum

Definition: Absolute Minimum

A function f has an **absolute minimum** (or **global minimum**) at $x = c$ if

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has an **absolute minimum** at $x = 0$ because

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The function

$$f(x) = -x^2$$

has an **absolute maximum** at $x = 0$ because

$$f(0) \geq f(x) \quad \text{for every } x \text{ in the domain of } f$$

Critical Number

Definition: Critical Number If f is a function and c is an element of the domain of f such that

$$f'(c) = 0$$

or

$$f'(c) \quad \text{does not exist}$$

then we say that c is a **critical number** of f.

Critical Number

Example: Find the critical numbers of $f(x) = x^2$

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First find the derivative:

$$f'(x) = 2x$$

Now set the derivative to zero and solve for x:

$$2x = 0 \quad \Rightarrow \quad x = 0$$

so we say that 0 is a **critical number** of $f(x) = x^2$.

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$$\cos x = 0 \quad \Rightarrow \quad x = \frac{\pi}{2}, -\frac{\pi}{2}, \pi, -\pi, \frac{3\pi}{2}, -\frac{3\pi}{2}, \ldots\ldots$$

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The critical numbers of $\cos x$ are:

$$\left\{ \pm\frac{n\pi}{2} : n = 1, 2, 3, \ldots \right\}$$

Question 1

Find the critical numbers of

$$f(x) = x^2 + 3x - 2$$

1. 1
2. $-\frac{3}{2}$
3. 2
4. $\frac{3}{2}$
5. 3
6. None of the above

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Question 2

Find the critical numbers of

$$f(x) = x - \ln x$$

1. 1
2. 0
3. 0,1
4. 0,-1
5. -1
6. None of the above

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Question 3

Find the critical numbers of

$$f(x) = \frac{x^3}{3} - x + 5$$

1. 1,3
2. 1,2
3. 1,-1
4. 1,-2
5. 2,3
6. None of the above

Question 3

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1. 1,3
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Extreme Value Theorem

Theorem: (Extreme Value Theorem) If f is continuous on a **closed** interval $[a, b]$ then f attains an absolute maximum $f(c)$ and absolute minimum value $f(d)$ at some values

$$c, d \in [a, b]$$

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Theorem: (Extreme Value Theorem) If f is continuous on a **closed** interval $[a, b]$ then f attains an absolute maximum $f(c)$ and absolute minimum value $f(d)$ at some values

$$c, d \in [a, b]$$

The interval $[a, b]$ has to be closed, that is, has to include its endpoints.

Closed Interval Method

To find the **absolute** maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

- Find the value of f at all critical numbers of f in $[a, b]$

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- Find the value of f at all critical numbers of f in $[a, b]$
- Find the values of f at the endpoints of the interval: $f(a)$ and $f(b)$
- The largest value produced by the previous two steps is the absolute maximum; the smallest is the absolute minimum of f on $[a, b]$.

Closed Interval Method

Example: Find the absolute max and min of $f(x) = 2x^2$ on $[-1, 2]$.

$f'(x)$ is $4x$, which exists everywhere on $[-1, 2]$. So, there are no values where $f'(x)$ fails to exist and no critical numbers of that kind.

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$f'(x)$ is $4x$, which exists everywhere on $[-1, 2]$. So, there are no values where $f'(x)$ fails to exist and no critical numbers of that kind.

$f'(x) = 0$ at $x = 0$, so 0 is a critical number of f.

The set of values of f at critical numbers consists of the single value $f(0) = 0$.

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The values of f at the endpoints are $f(-1) = 2$ and $f(2) = 8$.

Combining the values of f at critical numbers and endpoints, we have:

- $f(0) = 0$
- $f(-1) = 2$
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Combining the values of f at critical numbers and endpoints, we have:

- $f(0) = 0$
- $f(-1) = 2$
- $f(2) = 8$

The absolute max is 8, the absolute min is 0.

Question 4

Find the absolute minimum and maximum of

$$f(x) = x^2$$

on the interval $[-1, 3]$

1. $\min = 0, \max = 9$
2. $\min = 1, \max = 9$
3. $\min = 0, \max = 1$
4. $\min = 4, \max = 8$
5. $\min = \max = 9$
6. None of the above

Question 4

Find the absolute minimum and maximum of

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on the interval $[-1, 3]$

1. $\min = 0, \max = 9$
2. $\min = 1, \max = 9$
3. $\min = 0, \max = 1$
4. $\min = 4, \max = 8$
5. $\min = \max = 9$
6. None of the above

1.

Question 5

Find the absolute minimum and maximum of

$$f(x) = x^2$$

on the interval $[-2, 1]$

1. $\min = 0, \max = 1$
2. $\min = 1, \max = 4$
3. $\min = 4, \max = 9$
4. $\min = 0, \max = 9$
5. $\min = 0, \max = 4$
6. None of the above

Question 5

Find the absolute minimum and maximum of

$$f(x) = x^2$$

on the interval $[-2, 1]$

1. $\min = 0, \max = 1$
2. $\min = 1, \max = 4$
3. $\min = 4, \max = 9$
4. $\min = 0, \max = 9$
5. $\min = 0, \max = 4$
6. None of the above

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Question 6

Find the absolute minimum and maximum of

$$f(x) = x^2$$

on the interval $[1, 3]$

1. $\min = 0, \max = 9$
2. $\min = 0, \max = 1$
3. $\min = 1, \max = 4$
4. $\min = 1, \max = 9$
5. $\min = 4, \max = 9$
6. None of the above

Question 6

Find the absolute minimum and maximum of

$$f(x) = x^2$$

on the interval $[1, 3]$

1. $\min = 0, \max = 9$
2. $\min = 0, \max = 1$
3. $\min = 1, \max = 4$
4. $\min = 1, \max = 9$
5. $\min = 4, \max = 9$
6. None of the above

4.

Question 7

Find the absolute minimum and maximum of

$$f(x) = x - \ln x$$

on the interval $[1, 3]$

1. $\min = 1, \max = 1.19$
2. $\min = 1, \max = 1.31$
3. $\min = 0, \max = 1.31$
4. $\min = 1.19, \max = 1.3$
5. $\min = 0, \max = 1.19$
6. None of the above

Question 7

Find the absolute minimum and maximum of

$$f(x) = x - \ln x$$

on the interval $[1, 3]$

1. $\min = 1, \max = 1.19$
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3. $\min = 0, \max = 1.31$
4. $\min = 1.19, \max = 1.3$
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2.