Related Rates

The formula for the area of a circle is

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The formula expresses a relationship between the area of the circle A and its radius r

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Suppose we are told that the dimensions of the circle are changing over time.

Now, think of the area and radius as functions of the independent variable t:

r(t)	=	radius at time t
A(t)	=	area at time t

Related Rates

We can rewrite the area formula to reflect our new viewpoint that A and r are functions of t:

$$A(t) = \pi[r(t)]^2$$

The relationship between the area and radius has not been changed by the fact that both are time varying (that is, functions of t).

Related Rates

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Assuming that $A(t)$ and $r(t)$ are differentiable, we should be able to differentiate both sides of this expression with respect to t.

To do this, we use the technique of implicit differentiation.

Related Rates

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$$A(t)$$

we simply replace it by its derivative

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If we encounter a composite function, for example,

$$\pi \cdot [r(t)]^2$$

we apply the chain rule with inner function

$$g(t) = r(t)$$

Related Rates

In this case the outer function is

$$f(u) = u^2$$

so we apply the chain rule with

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so we apply the chain rule with

$$f(u) = u^2$$
$$g(t) = r(t)$$

The result is

$$\frac{d}{dt}[r(t)]^2 = f'(g(t)) \cdot g'(t)$$

or

$$2r(t)\frac{dr}{dt}$$

Related Rates

So we have differentiated both sides of the revised area equation

$$A(t) = \pi[r(t)]^2$$

which relates the area and radius of the circle, and obtained a new equation

$$\frac{dA}{dt} = 2\pi \cdot r(t)\frac{dr}{dt}$$

which relates the **rates of change** of the area and radius.

This is where the section title, **Related Rates** comes from.

Related Rates

The new equation says that if at some time t, the rate of change of the radius is

$$\frac{dr}{dt}$$

at that instant the rate of change of the area, dA/dt is

$$2\pi r(t)\frac{dr}{dt}$$

Note that we need both dr/dt and $r(t)$ to determine dA/dt.

Related Rates

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The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

Considering V and r as functions of time $V(t)$ and $r(t)$ and using implicit differentiation with respect to t, we get

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

Related Rates

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

This formula relates the time rate of change of the volume to the time rate of change of the radius.

Related Rates

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Considering A and r as functions of time $A(t)$ and $r(t)$ and using implicit differentiation with respect to t, we get

$$\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

Related Rates

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Considering A and r as functions of time $A(t)$ and $r(t)$ and using implicit differentiation with respect to t, we get

$$\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

This formula relates the time rate of change of the surface area of a sphere to the time rate of change of the radius.

Related Rates

If x and y are the sides of a right triangle, and z is the hypotenuse, the three lengths are related by the pythagorean formula

$$z^2 = x^2 + y^2$$

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Considering x, y and z as functions of time $x(t)$, $y(t)$, and $z(t)$, we can use implicit differentiation with respect to t to get

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

Related Rates

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$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

This formula relates the time rates of change of the sides of the triangle to each other.