Stewart Section 3.1

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Derivatives of Polynomials and Exponentials

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In practice, this is almost always how derivatives are actually evaluated.

You still need to know how to apply the definition, because most of the formulas assume that the derivative exists, and you usually need the definition to determine whether the derivative exists or not.

Derivatives of Polynomials and Exponentials

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for some real number c. The fact that x does not appear in the rule of assignment means that $f(x)$ is the same for all values of x.

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So we have our first result:

The derivative of a constant function is zero

The Power Rule

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This formula works for any n, positive or negative, whether n is an integer or not.

This formula is very important because power functions are often the building blocks of more complicated functions like polynomials.

The Constant Multiple Rule

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If you apply the definition of the derivative to this function, the result is

$$\frac{d}{dx}[c \cdot f(x)] = c\frac{d}{dx}f(x)$$

provided f is differentiable.

The Sum and Difference Rules

If f and g are differentiable functions, then the derivative of their sum is

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If f and g are differentiable functions, then the derivative of their difference is

$$\frac{d}{dx}(f-g)(x) = \frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

The Derivative of e^x

There are a number of ways to define the number e, the base of the natural logarithm and natural exponential functions.

The text defines e as the number which satisfies

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

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With this definition of e, you can apply the definition of the derivative to obtain the formula

$$\frac{d}{dx}(e^x) = e^x$$