What f' Says About f

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Note that if f' is continuous and it goes from positive to negative or from negative to positive on some interval, the intermediate value theorem says there has to be a point in the interval where $f'(x) = 0$. We will see that these points are the candidates for local maxima and minima.

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One way to remember this is to consider what having $f'' > 0$ on an interval says about f' on that interval - namely, that $f'(x)$ is *increasing* on that interval, so the slope of the tangent line is increasing.

If you picture a graph that is concave upward, the slope of the tangent line $f'(x)$ is increasing as x increases.

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Usually if f is a given function, it's antiderivative is denoted by F.