Stewart Section 2.6

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Tangents, Velocities, and Rates of Change

Definition: The **tangent line** to the curve $y = f(x)$ at the point $P = (a, f(a))$ is the line through P having slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

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The slope of the tangent line can also be expressed as

$$m = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

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We may define the velocity or instantaneous velocity $v(a)$ at time $t = a$ by the limit of this average velocity:

$$v(a) = \lim_{h \to a} \frac{f(a+h) - f(a)}{h}$$

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The average rate of change of y with respect to x over the interval $[x_1, x_2]$.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Tangents, Velocities, and Rates of Change

The *(instantaneous) rate of change of* y *with respect to* x *at* $x = x_1$ is defined by the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$