Sections 2.3

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Direct Substitution Property

If f is a **polynomial** or a **rational function** and a is in the domain of f, then

$$\lim_{x \to a} f(x) \quad = \quad f(a)$$

Theorem

If

$$f(x) \leq g(x)$$

when x is near a (except possibly at $x = a$), and

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

both exist, then

$$\lim_{x \to a} f(x) \quad \leq \quad \lim_{x \to a} g(x)$$

The Squeeze Theorem

If

$$f(x) \leq g(x) \leq h(x)$$

when x is near a (except possibly at $x = a$), and

$$\lim_{x \to a} f(x) \quad = \quad \lim_{x \to a} h(x) \quad = \quad L$$

then

$$\lim_{x \to a} g(x) \quad = \quad L$$

Problem 1

Suppose that for all values of x,

$$x^3 - 4x^2 + 6 \quad \leq \quad f(x) \quad \leq \quad x^4 - 3x^3 + 2x + 3$$

Find

$$\lim_{x \to 1} f(x)$$

Problem 1

Suppose that for all values of x,

$$x^3 - 4x^2 + 6 \quad \leq \quad f(x) \quad \leq \quad x^4 - 3x^3 + 2x + 3$$

Find

$$\lim_{x \to 1} f(x)$$

Solution:

The limit is 3 by the squeeze theorem.

Problem 2

Determine

$$\lim_{x \to 2} \frac{x^2 - 4}{|\, x - 2 \,|}$$

if the limit exists exists.

Problem 2

Determine

$$\lim_{x \to 2} \frac{x^2 - 4}{|\, x - 2 \,|}$$

if the limit exists exists.

Solution:

The limit does not exist because the left and right hand limits are different.

Problem 3

If

$$f(x) = \frac{5x^2 - 3}{x^2 + 6x + 2}$$

find

$$\lim_{x \longrightarrow 2} f(x)$$

if the limit exists exists.

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If

$$f(x) = \frac{5x^2 - 3}{x^2 + 6x + 2}$$

find

$$\lim_{x \to 2} f(x)$$

if the limit exists exists.

Solution:

2 belongs to the domain of f, so by direct substitution the limit is

$$\frac{17}{18}$$

Problem 4

If

$$f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ x^3 - 2x + 1 & x < 1 \end{cases}$$

find

$$\lim_{x \longrightarrow 1} f(x)$$

if the limit exists exists.

Problem 4

If

$$f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ x^3 - 2x + 1 & x < 1 \end{cases}$$

find

$$\lim_{x \longrightarrow 1} f(x)$$

if the limit exists exists.

Solution:

The limit is zero.

Problem 5

If

$$f(x) = \begin{cases} x^2 - 1 & x \geq 5 \\ x^3 - 2x + 1 & x < 5 \end{cases}$$

find

$$\lim_{x \longrightarrow 2} f(x)$$

if the limit exists exists.

Problem 5

If

$$f(x) = \begin{cases} x^2 - 1 & x \geq 5 \\ x^3 - 2x + 1 & x < 5 \end{cases}$$

find

$$\lim_{x \to 2} f(x)$$

if the limit exists exists.

Solution:

By direct substitution, the limit is 5.

Problem 6

If

$$f(x) = \frac{x^3 + x^2 - 4x - 4}{x + 2}$$

find

$$\lim_{x \to -2} f(x)$$

if the limit exists exists.

Problem 6

If

$$f(x) = \frac{x^3 + x^2 - 4x - 4}{x + 2}$$

find

$$\lim_{x \to -2} f(x)$$

if the limit exists exists.

Solution:

The limit is 4.