MTH141 Stewart Section 2.1

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The Tangent Problem

Given a function whose graph passes through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$, the average rate of change of f on the interval from x_1 to x_2 is

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

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We say that the slope of the tangent line is the *limit* of the slope of the secant line as $x_2 \to x_1$.

The Velocity Problem

The equation describing the number of meters an object falls in time t is

$$s(t) = 4.9t^2$$

The average velocity of the object from $t = 0.5$ to $t = 0.6$ is the distance the object falls in that length of time, divided by the length of time:

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The limit of this quantity as $t \to 0.5$ is called the instantaneous velocity at time $t = 0.5$. which is the limit of the slope of the secant line from $(x_1, f(x_1))$ to $(x_2, f(x_2))$.

Instantaneous Rate of Change

In general, the average rate of change of a function f over the interval from x to $x+h$ is

$$\frac{f(x+h)-f(x)}{h}$$

and the instantaneous rate of change is the limit of this quantity as $h \to 0$.

Sample Problem

Suppose you find yourself on the moon. Take out your bow and arrow, and shoot an arrow upward. If the arrow leaves the bow with an upward velocity of $58m/s$ and its height after t seconds is given by

$$h = 58t - 0.83t^2$$

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The average velocity during the time interval $[t, t+h]$ is given by

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Expanding all terms gives

$$\frac{58t + 58h - 0.83t^2 - 1.66th - 0.83h^2 - 58t + 0.83t^2}{h}$$

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Now collect like terms. Some cancellation occurs and the result is

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If we want the average over the time interval [0,3], then $t = 0$ and $h = 3$.
By substitution, the average velocity in m/sec is

$$58 - 1.66 \cdot 0 - 0.83 \cdot 3 = 58 - 2.49 = 55.51 m/s$$

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Note that the instantaneous velocity at time t is given by

$$\lim_{h \to 0} 58 - 1.66t - 0.83h = 58 - 1.66t$$

so at time $t = 3$, the instantaneous velocity is $58 - 1.66 \cdot 3 = 53 m/s$.