MA125 Stewart Section 1.3

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Transformations of Functions

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Transformations play an important role in many areas of mathematics, and calculus is one of them.

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Example: Suppose

$$f(x) = 3x + 4$$

The graph of f is a straight line with a slope of 3 and a (y) intercept of 4.

The fact that the slope is 3 means that the value of $f(x)$ or y increases by 3 units for each unit increase in x.

Translations

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Since we know $f(x) = 3x + 4$, we can expand the definition of y to

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The graph is now a straight line with a slope of 3 and a (y) intercept of 6.

So, defining a new function by adding 2 to f produced a function whose graph has the same shape as the original function, but shifted (or "translated") up 2 units.

Translations

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Even more generally, for any positive constant c,

$$y = f(x) + c$$

is a function whose graph has the same shape as the graph of f, but is shifted or translated c units up.

Translations

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There are also transforms that shift the graph to the left or right, while preserving the shape of the graph.

The only difference is that we add the constant to x, **then** apply the rule of assignment. Using the same function as before, $f(x) = 3x + 4$, we will add 2 to x, then evaluate f:

$$y = f(x + 2) = 3(x + 2) + 4$$

or

$$y = 3x + 10$$

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The only difference is that we add the constant to x, **then** apply the rule of assignment. Using the same function as before, $f(x) = 3x + 4$, we will add 2 to x, then evaluate f:

$$y = f(x+2) = 3(x+2) + 4$$

or

$$y = 3x + 10$$

Translations

You can verify that the new function

$$y = 3x + 10$$

has a graph with the same shape as the original function,

$$y = 3x + 4$$

but shifted or translated 2 units to the left.

Translations

Many people find it confusing that although we added positive 2 to x, the graph shifted to the left, the negative direction.

One device people use to get the direction correct is to remember that the graph of

$$y = f(x+2)$$

looks the same as the graph of

$$y = f(x)$$

but with the vertical axis shifted two units to the **right**.

Vertical and Horizontal Translations - Summary

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$y = f(cx)$ compresses the graph of f **horizontally** by a factor of c

$y = f\left(\frac{x}{c}\right)$ stretches the graph of f **horizontally** by a factor of c

Reflecting - Summary

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$y = -f(x)$ reflects the graph of f about the x axis

$y = f(-x)$ reflects the graph of f about the y axis

Combinations of Functions - Sums

If f and g are functions with domains A and B, respectively, then we can define a new function

$$(f+g)(x) = f(x) + g(x) \quad \text{domain} = A \cap B$$

Combinations of Functions - Differences

If f and g are functions with domains A and B, respectively, then we can define a new function

$$(f - g)(x) = f(x) - g(x) \quad \text{domain} = A \cap B$$

Combinations of Functions - Products

If f and g are functions with domains A and B, respectively, then we can define a new function

$$(fg)(x) = f(x) \cdot g(x) \quad \text{domain} = A \cap B$$

Combinations of Functions - Quotients

If f and g are functions with domains A and B, respectively, then we can define a new function

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{domain} = \{x \in A \cap B \mid g(x) \neq 0\}$$

Combinations of Functions - Composition

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The domain of $f \circ g$ is the set of all x in the domain of g for which $g(x)$ is in the domain of f.