Question 1

Find the domain of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by:

$$f(x) = \frac{1}{\sqrt{x-4}}$$

A. $\mathbb{R}$

B. $(-\infty, -4)$

C. $(-\infty, 4)$

D. $\mathbb{R} \setminus 4$

E. $(4, \infty)$

F. $(0, \infty)$

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E. $(4, \infty)$

F. $(0, \infty)$

The domain of f is: E. $(4, \infty)$ or $\{x : x > 4\}$

Question 1

We need to find all values of x for which

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is a real number. This is happens when $\sqrt{x-4}$ is real and positive.

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This in turn is true when $x > 4$, so the domain is

$$D_f = \{x : x > 4\} \quad = \quad (4, \infty)$$

Question 2

Find the **range** of the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \frac{1}{\sqrt{x-4}}$$

A. $\mathbb{R}$

B. $(-\infty, 4)$

C. $(-\infty, 0)$

D. $\mathbb{R} \setminus 4$

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E. $(4, \infty)$

F. $(0, \infty)$

The range of f is: F. $(0, \infty)$

Question 2

Usually you can get an idea of the range by evaluating f for the smallest and larges values in the domain, $(4, \infty)$ in this case. When x becomes very large, $\sqrt{x-4}$ becomes very large, and

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becomes very small, but always greater than zero.

When x is close to 4, $f(x)$ is very large and positive, tending to ∞ as $x \to 4$. So the range is:

$$R_f = \{x : 0 < x < \infty\} \quad = \quad (0, \infty)$$

Question 3

Find the domain of the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \frac{1}{\sqrt{1 + x^2}}$$

A. $\mathbb{R}$

B. $(-\infty, -1)$

C. $(-\infty, 1)$

D. $\mathbb{R} \setminus -1$

E. $(-1, \infty)$

F. $(1, \infty)$

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A.	$\mathbb{R}$	D.	$\mathbb{R} \setminus -1$
B.	$(-\infty, -1)$	E.	$(-1, \infty)$
C.	$(-\infty, 1)$	F.	$(1, \infty)$

The domain of f is: A. $\mathbb{R}$ or $(-\infty, \infty)$

Question 3

The domain of this function is the set of real numbers for which

$$\sqrt{1+x^2} > 0$$

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is real and positive, but since the smallest value x^2 can assume is zero, $1+x^2$ is always positive.

Consequently, the domain is all real numbers,

$$D_f = \mathbb{R} \quad = \quad (-\infty, \infty)$$

Question 4

Find the **range** of the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \frac{1}{\sqrt{1 + x^2}}$$

A. $\mathbb{R}$

B. $(-1, 0)$

C. $(-\infty, 1)$

D. $(-1, 1)$

E. $(-1, \infty)$

F. $(0, 1]$

Question 4

Find the **range** of the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \frac{1}{\sqrt{1 + x^2}}$$

A. $\mathbb{R}$

B. $(-1, 0)$

C. $(-\infty, 1)$

D. $(-1, 1)$

E. $(-1, \infty)$

F. $(0, 1]$

The range of f is: F. $(0, 1]$

Question 4

The largest value of $f(x)$ occurs when the denominator is as small as possible, which happens when $x = 0$. In this case

$$f(x) = \frac{1}{\sqrt{1 + x^2}} = \frac{1}{\sqrt{1}} = 1$$

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$$f(x) = \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1}} = 1$$

For values of x larger than zero in absolute value, $f(x)$ is between 0 and 1, so the range is:

$$R_f = \{x : 0 < x <\leq 1\} \quad = \quad (0, 1]$$

Question 5

Find the domain of the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \frac{1}{x^2 - 4}$$

A. $\mathbb{R}$

B. $\mathbb{R} \setminus \{-2, 2\}$

C. $(-\infty, -2) \cup (2, \infty)$

D. $\mathbb{R} \setminus \{2\}$

E. $(-2, 2)$

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D. $\mathbb{R} \setminus \{2\}$

E. $(-2, 2)$

F. $(2, \infty)$

The domain of f is: B. $\mathbb{R} \setminus \{-2, 2\}$

Question 5

The domain of a rational function is the set of real numbers for which the denominator is not zero. So we have to exclude values of x that satisfy

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Factoring the left hand side as $(x-2)(x+2)$, we see that the values we have to exclude are 2 and -2.

$$D_f = \mathbb{R} \setminus \{-2, 2\} \quad = \quad (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

Question 6

Find the **range** of the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \frac{1}{x^2 - 4}$$

A. $\mathbb{R}$

B. $(-\infty, 0)$

C. $(-\infty, -1/4) \cup (0, \infty)$

D. $(0, \infty)$

E. $(-1/4, \infty)$

F. $(-1/4, 0)$

Question 6

Find the **range** of the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \frac{1}{x^2 - 4}$$

A. $\mathbb{R}$

B. $(-\infty, 0)$

C. $(-\infty, -1/4) \cup (0, \infty)$

D. $(0, \infty)$

E. $(-1/4, \infty)$

F. $(-1/4, 0)$

The range of f is: C. $(-\infty, -1/4) \cup (0, \infty)$

Question 6

The graph of this function is: