Example 1

A technique for handling expressions with radicals is called *rationalization*.

Suppose we want to find the difference quotient for $f(x) = \sqrt{x}$:

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

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The idea is to multiply the numerator

$$\sqrt{x+h} - \sqrt{x}$$

by its *conjugate*:

$$\sqrt{x+h} + \sqrt{x}$$

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In this case $a = \sqrt{x+h}$ and $b = \sqrt{x}$, so we have

$$(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x}) \;=\; (\sqrt{x+h})^2 - (\sqrt{x})^2$$

Then

$$(\sqrt{x+h})^2 - (\sqrt{x})^2 \;=\; (x+h) - x \;=\; h$$

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Now multiply by 1 in a carefully chosen form:

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The result is

$$\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h\cdot(\sqrt{x+h}+\sqrt{x})}$$

Example 2

Find

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 16} - 4}{x^2}$$

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Again use the rationalization via the conjugate:

$$\frac{\sqrt{x^2+16}-4}{x^2} = \frac{\sqrt{x^2+16}-4}{x^2} \cdot \left(\frac{\sqrt{x^2+16}+4}{\sqrt{x^2+16}+4} \right)$$

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$$\frac{(\sqrt{x^2+16})^2-4^2}{x^2(\sqrt{x^2+16}+4)}=\frac{1}{\sqrt{x^2+16}+4}$$

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$$\frac{(\sqrt{x^2+16})^2-4^2}{x^2(\sqrt{x^2+16}+4)} = \frac{1}{\sqrt{x^2+16}+4}$$

By direct substitution, the limit as $x \to 0$ is $1/8$.

Question 1

Find

$$\lim_{x \to 16} \frac{4 - \sqrt{x}}{16x - x^2}$$

(if the limit exists)

1. -1/16
2. 1/128
3. 3/48
4. 1/64
5. does not exist
6. None of the above

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2. The limit is 1/128

Question 2

Find

$$\lim_{x \to -4} \frac{\sqrt{x^2+9}-5}{x+4}$$

(if the limit exists)

1. -3/5
2. 2/5
3. 3
4. -4/5
5. does not exist
6. None of the above

Question 2

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$$\lim_{x \to -4} \frac{\sqrt{x^2+9}-5}{x+4}$$

(if the limit exists)

1. -3/5
2. 2/5
3. 3
4. -4/5
5. does not exist
6. None of the above

4. The limit is -4/5

Question 3

Find

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}$$

(if the limit exists)

1. -1/3
2. 1/2
3. 3
4. 1
5. does not exist
6. None of the above

Question 3

Find

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$$

(if the limit exists)

1. -1/3
2. 1/2
3. 3
4. 1
5. does not exist
6. None of the above

2. The limit is 1/2